PRELIMINARY TURBINE DESIGN FOR THERMAL CYCLE FEASIBILITY

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by

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ABSTRACT

Preliminary Turbine Design for Thermal Cycle Feasibility

David A. Potter

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Mathematical models of an axial flow turbine and a radial inflow turbine were developed. Utilizing these models, a computer program was written such that given a thermal cycle and a minimum of input variables, the necessary turbine design calculations are performed. An extensive output is provided such that the designer may utilize this output in order to evaluate the practical feasibility of the given thermal cycle.

The Zener sea cycle is used as a test case with axial and radial designs examined for feasibility.

The computer program is written in the Fortran IV language specifically to be run on the Interdata 70 computer.

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NOTATION

		UNITS
A_h	Blade cross sectional hub area	ft^2
A_f	Flow area	ft^2
A_{ann}	Annulus	ft^2
c	Blade chord length	ft^2
C_p	Specific heat at constant pressure	BTU/^oF
C_v	Specific heat at constant volume	BTU/^oF
D	Hydraulic diameter	ft
g_o	Gravitational constant	32.174 ft lbm/lbf sec^2
h	Enthalpy	BTU/lbm
$I_{\bar{\theta}}$	Moment of Inertia in tangential direction	ft^4
$I_{\bar{x}}$	Moment of Inertia in Axial direction	ft^4
J	Conversion factor	778 ft-lbf/BTU
L_w	Windage loss	BTU/lbm
L_s	Stator loss	BTU/lbm
L_r	Rotor loss	BTU/lbm
L_c	Clearance loss	BTU/lbm
$\dot{m}$	Mass flow rate	lbm/sec
M	Mach Number	Non-Dim
M_θ	Bending moment in tangential direction	ft-lbf
M_x	Bending moment in axial direction	ft-lbf

n	Number of axial stages	Non-Dim
N_s	Specific speed	Non-Dim
O	Throat opening	feet
P	Pressure	lbf/in^2
Q	Volumetric flow rate	ft^3/sec
R	Universal gas constant	BTU/lbm ^oR
$\mathcal{R}$	Reaction	Non-Dim
r	Radius	feet
R_e	Reynold's Number	Non-Dim
RPM	Shaft rotative speed	Rev/min
S	Spacing	feet
s	Entropy	BTU/lbm ^oR
T	Temperature	Deg Rankine
t	Blade height	feet
U	Rotor tangential speed	ft/sec
V	True fluid speed	ft/sec
W	Relative fluid speed	ft/sec
x	Axial distance	feet
α	True fluid flow angle from axial	Deg
β	Relative fluid flow angle from axial	Deg
γ	Ratio of C_p/C_v	Non-Dim
ζ_o	Primary loss factor	Non-Dim
ζ_1	Primary loss factor corrected for aspect ratio	Non-Dim

ζ_a	Primary loss factor corrected for AR & R_e	Non-Dim
ζ	Slip factor	Non-Dim
η	Efficiency	Non-Dim
θ	Tangential angle	Deg
λ	Stage loading coefficient	Non-Dim
μ	Static viscosity	lbm/hr ft
ν	Kinematic viscosity	ft^2/hr
Δ	Differential function	Non-Dim
σ	Solidity	Non-Dim
σ_c	Centrifugal stress	lbf/in^2
π	Constant	3.1415927
ρ	Density	lbm/ft^3
ϕ	Flow coefficient	Non-Dim
Ψ	Work coefficient	Non-Dim
ω	Angular rotative speed	Radians/sec

SUBSCRIPTS

o	Stagnation conditions
o1	Machine inlet stagnation condition
o2	Machine outlet stagnation condition
o2s	Isentropic outlet stagnation condition
i	Individual flow path
t	Total flow paths
θ	Tangential direction
X	Axial or meridional direction

m	Mean
h	Hub
t	Tip
f	Fluid
tt	Total to total
ts	Total to static
mat.	Material
max	Maximum
Axial	
1	Rotor inlet
2	Rotor outlet
Radial	
1	Stator inlet
2	Stator outlet
3	Rotor inlet
4	Rotor outlet

INTRODUCTION

The thermal cycle is the basic tool in use today to convert the energy contained in a source or fuel into useful power. Cycles using geothermal energy, cycles powered by solar energy, and cycles powered by the thermocline existing in the oceans, are just a few of the proposed cycles being studied as possible sources of power for the future. In the search for new and plentiful energy sources in the years to come, new and different thermal cycles will be proposed.

The evaluation of a given thermal cycle as to its practical feasibility involves a detailed study in which not only the energy source, but the hardware needed to convert that energy into useful power, are investigated. In such a study, the engineer must concern himself with three main areas: converting the energy source to heat, transferring that heat energy to a working medium, and then utilizing the increased energy of the working medium to produce useful work or power. It is with this last facet of the problem that this thesis is concerned.

BACKGROUND

1.1 Thermal Cycles

Thermal cycles come in all sizes and shapes. They are used not only to convert heat energy to useful work, but to transfer heat energy from one location to another, as in a refrigeration unit. The difference between the two cycles, other than hardware, lies in the fact that in a work-orientated thermal cycle energy is supplied in the form of heat or fuel and work is the end result; while in a heat transfer or refrigeration cycle, work is supplied as an input and the end result is the transfer of heat energy. While both types of cycles operate under the same principles, it is the former type which is of great concern in the search for means to supply society's growing energy demands, and which will be the only one considered from hereon.

Thermal cycles of the heat to work type can be categorized as either open or closed cycles. In the open cycle, the energy of the source (fuel) is transferred to a working medium in the form of heat, whereupon this increased energy of the working medium is utilized to produce useful work. At the conclusion of the open cycle, the working medium is exhausted from the machine. In a closed cycle the process is essentially the same; however, the working medium is retained in the cycle after producing work and is reintroduced to the source where it again increases its energy state with the addition of heat. There are, of course, many variations

and combinations of the above cycles using many different working mediums at many different temperatures and pressures.

1.2 Turbomachinery

One problem common to all thermal cycles, open or closed, is that of finding an efficient means to convert the energy of the working medium into useful work. Without considering unproven schemes, such as magnetohydrodynamics, there are two basic classes of machines in use today with which this can be accomplished: machines which extract the energy from the working medium as a change in its potential energy, and those machines which extract the energy as a change in the kinetic energy of the working medium. Without discussing the relative merits of either type of machine, it is sufficient to state that the second class of machine, the turbomachine, is a viable alternative and one which should be considered in any thermal cycle feasibility study.

Turbomachines, or more specifically, turbines used in thermal cycles today, can be divided into two categories: those in which the working fluid flows through the machine in a nearly constant axial direction parallel to the centerline of the machine; and those in which the working fluid flows in a direction radial to the centerline of the machine, either inward or outward. The choice as to which type of machine is best for a certain set of conditions is a difficult one, depending on a large number of variables external

to the cycle itself, such as weight, cost, and operating load. This will be discussed more fully in following sections.

1.3 The Design Sequence

In determining the technical feasibility and practical value of a given cycle, at the preliminary design stage, the engineer must follow an iterative process. One logical sequence would be to:

1) Assume a working medium and a set of states for the medium (most commonly the energy extraction process is done in a gaseous state).
2) Assume certain hardware restraints and a flow rate for the working fluid.
3) Design the required hardware (if not available commercially).
4) Evaluate the necessary hardware and if not satisfactory return to step 1.

This process is repeated several times until the hardware conforms to the engineer's definition of feasibility, or until some prechosen parameters are optimized. The problem here lies in the fact that the calculations necessary to design the hardware, although relatively straightforward, are numerous and time-consuming. For a given set of cycle conditions, there are often many options open to the designer, all of which will result in different hardware and all of which will meet the design criteria. The choices between axial or radial

turbines, and between turbines or reciprocating machines, are good examples of this. There is a need, therefore, for the designer or engineer to quickly translate his cycle requirements into specific hardware sizes and characteristics in order to logically choose between the options available.

1.4 Optimization

Feasibility and optimization are two words whose definitions can be different for every different cycle considered. In a given situation it may be necessary to maximize efficiency, while in another situation, the design may be constrained such that weight or volume or cost must be minimized. As for turbines, in some applications an axial flow unit might be chosen for its high efficiency, while in others a radial machine with its lower fabrication cost might be best. Feasibility also has different meanings in most different contexts. In one instance it might mean technically possible, while in another it might mean financially profitable. In order to answer these questions, the engineer or designer must first define feasibility and then decide upon which, if any, parameters are to be optimized within the feasibility constraints. Having done this, he may then evaluate the hardware designs and make a logical choice between competing designs.

1.5 Purpose

Within this framework then, the purpose of this thesis is to bring together the various concepts of turbine design;

to develop a mathematical model; and to computerize that model in such a way that, given a minimal amount of information about the cycle and the machine constraints, a set of turbine hardware parameters will be provided that meet the requirements of the preliminary design stage. These may then be used in the design sequence mentioned previously to determine feasibility. No attempt will be made to optimize within the computer program, but rather to provide information from which a suitable optimization criteria can be evaluated. For this reason no attempt will be made to choose between an axial or a radial turbine. Both will be developed simultaneously and information will be provided on both types. The program will be capable of handling multiple inputs for the same cycle in one computer run.

It is hoped that by utilizing this program the design sequence may be shortened considerably. This will be true especially if use is made of other computer programs to calculate heat exchanger parameters. In doing so, the designer will be able to study more completely the possibilities of a given cycle at the preliminary design stage, and thus develop a more suitable final design.

DEVELOPMENT OF THE MATHEMATICAL MODEL

2.1 General

A design philosophy is a necessary ingredient to any worthwhile design. This philosophy may not always be explicitly stated; however, it is always present in a good design. It is a means whereby the designer is able to make logical and, of most importance, consistent decisions during the course of the design sequence. This is especially true at the preliminary design stage where decisions are made which constrain the course of the design throughout.

The philosophy followed in compounding the mathematical models described in the following sections and in the actual programming itself can be stated as accurate and simple. Both of these qualities apply to any work done at a preliminary design stage. Accuracy is required because the designs have now passed beyond the back of the envelope stage; and, although detailed design is not required yet, the results must be accurate enough so that detailed design can begin by using them as a basis. Simplicity is a must because at the preliminary stage, neither the time nor the money are usually available to carry out the extensive calculations of a detailed design. That is not the purpose of the preliminary design. One last idea is also followed in the handling of certain relationships; that in the absence of accurate and simple relationships to cover a certain situation, traditional and reasonable approximations are assumed. With the lack of

specific data in certain areas this borders on an educated guess, but then again this is what the designer is paid for.

As to the development of the models themselves, they can be divided into three main areas: representation of the working medium itself, representation of the axial turbine, and representation of the radial turbine. They will be explained in that order.

It is assumed that the machines described by the models can be built of conventional materials and that the axial blading has no twist and is set in a purely radial direction. It is further assumed that internally cooled blades or rotors are not utilized.

All computations are in English units with the notation as explained on pages 8 to 11. Calculations are carried through to a precision consistent with the accuracy of the Interdata 70 computer (six significant figures).

2.2 The Working Medium

2.2.1 Equation of State

Two assumptions concerning the working medium are basic but should be stated for clarity. It is assumed that the working medium is a fluid and that it enters the machine in a gaseous state; this precludes the design of hydraulic turbines. The second assumption is that the inlet and exit states of the fluid are known and that the designer has the properties of the fluid at these states available as input.

In order to accurately represent the fluid within the

machine, an equation of state for the fluid had to be picked. The perfect gas model was chosen over the other equations of state for its flexibility, its accuracy, its wide acceptance, and its compatibility with the standard one dimensional compressible flow relationships. It is therefore assumed that in the region under consideration, the fluid can be represented by the perfect gas model and that the effects of velocity can be represented by the standard one dimensional compressible flow relationships.

To make this assumption more realistic, it is necessary to calculate the different constants in the region under consideration rather than use traditional or tabulated values. One method to achieve this is to use data for an isentropic expansion in the region and to calculate the constants from this. Since the designer already has calculated the point in order to determine his outlet conditions, it was decided to use an isentropic expansion across the machine for the reference points. Thus from points 1 to 2s,

$$C_p = (h_{o2s}-h_{o1})/(T_{o2s}-T_{o1})$$

Using the Gibbs equation and the perfect gas law, the following commonly used relationship for an isentropic expansion may be derived:

$$(P_{o2s}/P_{o1}) = (T_{o2s}/T_{o1})^{\gamma/(\gamma-1)}$$

where $\gamma = C_p/C_v$

Solving this equation for γ:

$$\gamma = \ln(P_{o2s}/P_{o1})/(\ln(P_{o2s}/P_{o1}) - \ln(T_{o2s}/T_{o1}))$$

Thus $C_v = C_p/\gamma$

and $R = C_p - C_v$.

2.2.2 Flow Rate

In describing a particular turbomachine, the mass flow rate of the machine has a large effect on its design. For a particular cycle, the mass flow rate to achieve a given output can be easily calculated using the steady flow energy equation. It may not be advantageous, however, to achieve this output on only one machine, but rather to utilize several machines, connected in parallel, to achieve the given output. The mass flow rate through a given machine, with no allowance for leakage, will be:

$$\dot{m}_{individual} = \dot{m}_{total} \ / \ \#\ \text{units in parallel}$$

2.2.3 Compressible Flow Relations

The two parameters necessitating the use of the one dimensional flow relations are the calculation of Mach number and the calculation of flow area. Using these relationships, and given the stagnation or total temperature and the velocity of a fluid at a point, the Mach number and flow area may be easily calculated. From the definition of total temperature and Mach number:

$$T = T_o - V^2/2g_oJC_p$$

$$M = V/(\gamma g_o R T)^{\frac{1}{2}}$$

In its common form:

$$\dot{m}_i (T_o)^{\frac{1}{2}}/P_o A_f = M(\gamma g_o/R)^{\frac{1}{2}}/(1+(\gamma-1/2)M^2)^{(\gamma+1)/2(\gamma-1)}$$

Thus $\quad A_f = (\dot{m}_i (T_o)^{\frac{1}{2}}/P_o) \times f(\gamma, g_o, R, M)$

$$A_f = (\dot{m}_i (T_o)^{\frac{1}{2}}/P_o) \times (\text{flow function})$$

where:

$$\text{Flow function} = (1+(\gamma-1/2)M^2)^{(\gamma+1)/2(\gamma-1)}/M\ (\gamma g_o/R)^{\frac{1}{2}}$$

2.3 Axial Turbine

2.3.1 General

The first step in the design sequence involves deciding the general configuration of the machine. Is the hub radius, the mean radius, the blade tip radius or some combination of these to be maintained constant throughout the machine? Wilson (1) recommends that the mean radius be maintained constant, while Carmichael (2) recommends hub radius be kept constant at the preliminary design stage. Constant mean radius was chosen for the reason that the effect on turbine performance between the two is small, and calculations at a later stage will be simplified with a constant mean radius. Following this assumption, mean conditions are calculated first. As the velocity triangles and other blade parameters vary by stage throughout the machine and as the extremes are represented by the first and last stages, only the first and last stages will be calculated in detail with the other

stages assumed to lie in between these values. Finally, machine parameters which depend upon the previous calculation, such as total efficiency and machine length, will be made. It is also assumed at this point that the machine is a full admission turbine operated under a constant load; both assumptions are consistent with stationary power plant operation.

2.3.2 Required Output

By a process of elimination, the list of quantities which is sufficient to describe the turbine at the preliminary design stage was narrowed to the following. Many other quantities could have been included; however, all major parameters are included, and those omitted may simply be calculated from those given.

1)	First and Last stage	Tip Diameters
2)	"	Blade Height
3)	"	Aspect Radios
4)	"	Number of Blades
5)	"	Efficiency (Total to total)
6)	"	Velocity Triangles
7)	Machine	Critical Mach number
8)	"	Efficiency (Total to total)
9)	"	Critical Stresses
10)	"	Length

In describing the velocity triangles for a given stage, three locations for both the stator and rotor entrance triangles are described: hub, mean and tip.

2.3.3 Input

In listing the factors which are generally used to describe a turbine stage, it was found that they could be categorized in three groups: those which describe the mass flow rate; those which describe the work accomplished per pound of fluid per stage; and those which describe the performance of a given stage. Each facet is necessary for the preliminary design.

The mass flow rate for the machine, and thus each stage, (since the stages are assumed to be in series with no bleed-offs) can be described by the total mass flow rate for the cycle and the number of turbines in parallel, as described in section 2.2.2.

The work per pound of fluid per stage can be simply represented also. Making the assumption that the work accomplished per stage is equal for all stages (2), the individual stage work can be represented by the total work or enthalpy drop for the machine and the number of stages.

The parameters used to describe stage performance are not so straightforward. They can be classed as either primary or secondary, depending upon the strength of their influence upon performance. Primary factors are generally associated with the geometry of the velocity triangles, and

if the deviation is assumed negligible, the geometry of the blade itself. Secondary factors have a lesser effect on efficiency and are related to the likes of clearance, aspect ratio, and other mainly mechanical factors. In order to maintain a simple machine only primary factors will be specified as an input, with a logic developed for the determination of reasonable secondary factors based on the given primary factors.

As primary factors, Carmichael (2) recommends reaction, $\mathcal{R}$, and inlet flow angle, α_1. Craig and Cox (3) prefer to use the stage loading parameter, Ψ, also called the work coefficient, and the flow coefficient, Φ. λ is another widely used parameter (4). Comparing the definitions of these parameters,

$$\mathcal{R} = 1 - (V_{\theta 1} + V_{\theta 2})/2U$$

$$\text{Tan } (\alpha_1) = V_\theta / V_x$$

$$\Psi = \Delta U V_\theta / U^2$$

$$\phi = V_x / U$$

$$\lambda = U/(V_{\theta 1} - V_{\theta 2})$$

it can be seen that there are only four independent variables. For reasons explained in a later section, one of these can be eliminated; thus only three independent variables will describe the shape of the velocity triangles.

The three variables chosen for use here are $\mathcal{R}$, U , and α_1. Reaction was chosen for its compatibility with the Euler work equation which will be used later to solve for a mean radius. Given this mean radius and an RPM, the second variable, U, can be specified. The last variable, α_1, was chosen as it would have to be calculated anyway during the course of the design.

The input for the axial turbine can be summarized as:

1) Mass flow rate
2) Number of units in parallel
3) Number of stages per unit
4) Shaft revolutions per minute
5) Reaction at mean conditions
6) α_1 at mean conditions
7) Cycle conditions of section 2.2.1.

2.3.4 Mean Conditions

The starting point for the mean conditions is with the simultaneous solution of the Euler work equation and the reaction equation. It is assumed that the fluid enters the stator only in an axial direction, with no tangential component. This is a common assumption (4) and is made for several reasons. One argument is that the turbine works by extracting energy from the fluid; thus any energy left in the fluid at rotor exit (stator inlet) in the form of vorticity (tangential velocity) is unavailable for extraction, and thus contributes

to a loss. Using this assumption, the Euler equation applied to mean conditions across the rotor becomes

$$\frac{\Delta h_o}{n} = \omega r_m V_{\theta 1m}/g_o J$$

$$\text{where } \Delta h_o = h_{o1} - h_{o2}$$

The reaction equation, also applied at mean conditions across the rotor is simplified to

$$\mathcal{R}_m = 1 - V_{\theta 1m}/2\omega r_m.$$

The simultaneous solution of these equations for r_m and $V_{\theta 1m}$ with $\omega = \pi RPM/30$ gives

$$r_m = (\Delta h_o g_o J 450/\pi^2 RPM^2 n(1-\mathcal{R}_m))^{\frac{1}{2}}$$

$$V_{\theta 1m} = r_m \pi RPM(1-\mathcal{R}_m)/15.$$

Substituting in the appropriate constants,

$$r_m = 1068.247(\Delta h_o/n(1-\mathcal{R}_m))^{\frac{1}{2}}/RPM$$

and

$$V_{\theta 1m} = 223.7331(\Delta h_o(1-\mathcal{R}_m)/n)^{\frac{1}{2}}.$$

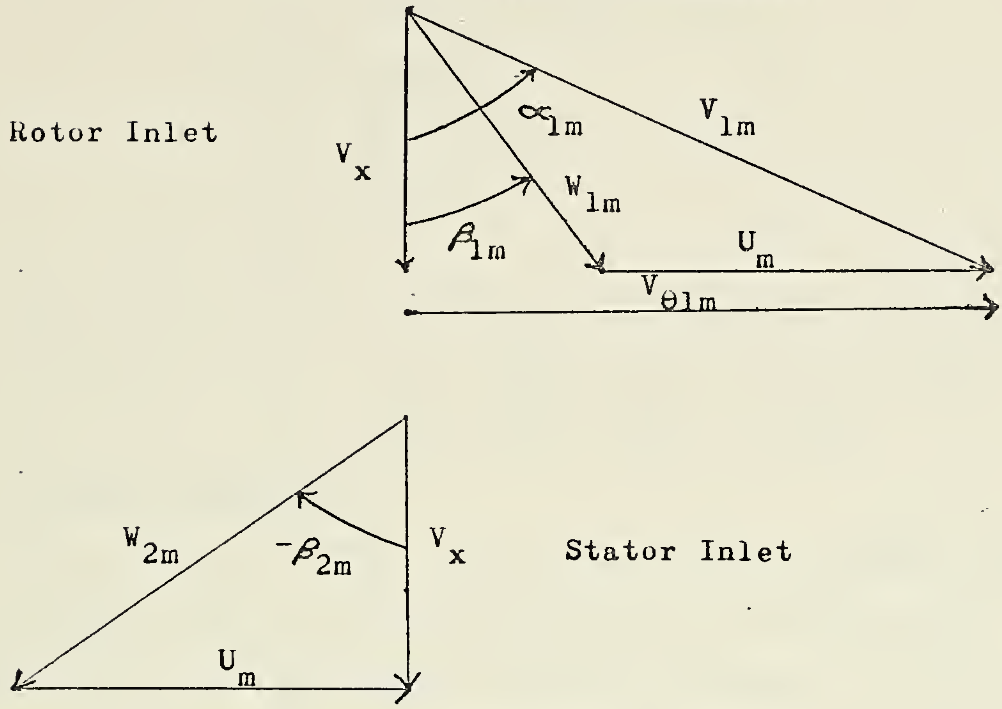
In order to prevent the solution from expanding at $\mathcal{R}_m=1$, $\mathcal{R}_m$ must be limited to less than 0.99.

The rotor speed can be calculated as

$$U_{1m} = U_{2m} = U_m = \pi RPM\ r_m/30.$$

Using the trignometric relationships, Figure 1, and the α_1 which was given as an input, the rest of the velocity triangle for the rotor inlet can be calculated.

Axial Turbine Velocity Triangles



<u>Figure 1</u>

$$V_{1m} = V_{\theta 1m}/\mathrm{Sin}(\alpha_{1m})$$

$$V_x = V_{\theta 1m}/\mathrm{Tan}(\alpha_{1m})$$

$$\beta_{1m} = \mathrm{ArcTan}(\mathrm{Tan}(\alpha_{1m})\ (1-1/(2-2R_m)))$$

$$W_{1m} = V_x/\mathrm{Cos}(\beta_{1m}).$$

Making the assumption that the axial velocity is constant throughout the machine as was stated before, the tangential velocity at rotor outlet is zero; thus the velocity triangle for the stator inlet (rotor outlet) can be calculated:

$$\beta_{2m} = \mathrm{ArcTan}(\mathrm{Tan}(\alpha_{1m})/2(1-R_m))$$

$$W_{2r} = V_{\theta 1m}/\cos(\beta_{2m})\tan(\alpha_{1m})$$

$$\alpha_{2m} = V_{\theta 2m} = 0$$

$$V_{x1} = V_{x2} = V_x$$

$$V_{2m} = V_x.$$

The total gas path deflection for the rotor is the sum of the inlet and outlet angles ($\beta_{1m} + \beta_{2m}$); while for the stator it is just α_{1m}, as the inlet angle is zero.

2.3.5 Stage Conditions

In solving for stage conditions it can be assumed that the extreme values are represented by the first and last stages, with all other stages falling in between. If the variance is assumed linear, the dimensions and velocities can be calculated for any stage with reasonable accuracy. This assumption also allows the calculation of machine efficiency as the mean of first and last stage efficiencies. Based on these assumptions, the following calculations can be made for the first and last stages.

Using the one dimensional compressible flow relationships described in section 2.2.3, the flow area can be calculated for a given velocity and stagnation temperature. The flow area and annulus area are related as shown in Figure 2.

$$Area_{ann} = Area_{flow}/\cos(\alpha).$$

Using the definition of annulus area and the mean radius, the blade height may be found.

Axial Turbine Flow Area

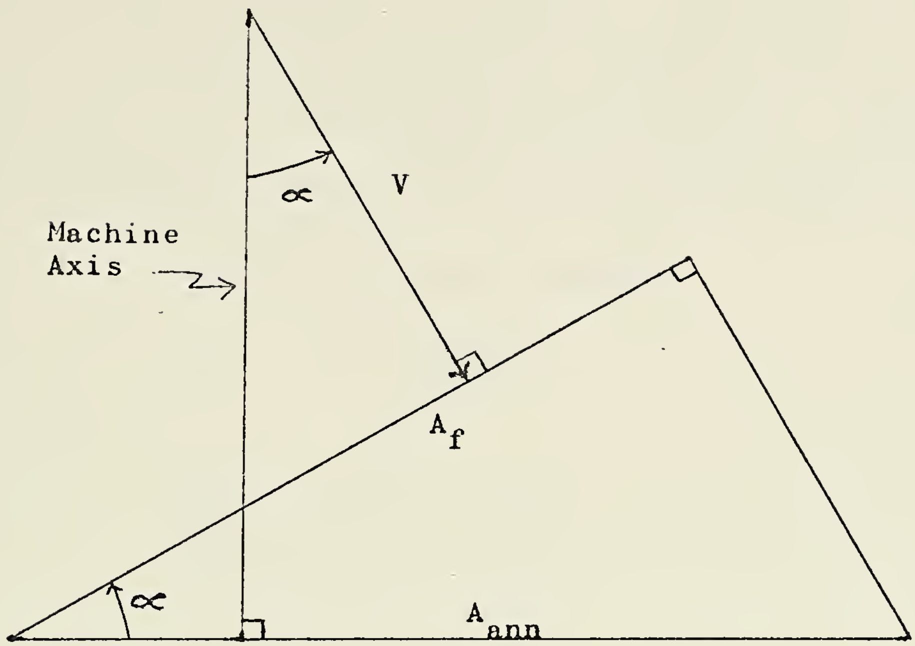


Figure 2

$$\text{Area}_{ann} = \pi(r_t^2 - r_h^2)$$

$$\text{"} \quad = \pi(r_t - r_h)(r_t + r_h)$$

$$\text{"} \quad = \pi t 2 r_m$$

Blade height $= t = \text{Area}_{ann}/2\, r_m \pi$

where $r_m = (r_t + r_h)/2$

The tip and hub radii are found by respectively adding or subtracting one half of the blade height to the mean radius.

The next step involves making the assumption that a streamline of fluid has no appreciable curvature in the radial

direction as it moves through the machine. This is not completely correct, but for small wall curvatures it can be considered accurate (1). This assumption means that a segment of fluid, as shown in Figure 3, must be in radial equilibrium.

RADIAL EQUILIBRIUM FORCES

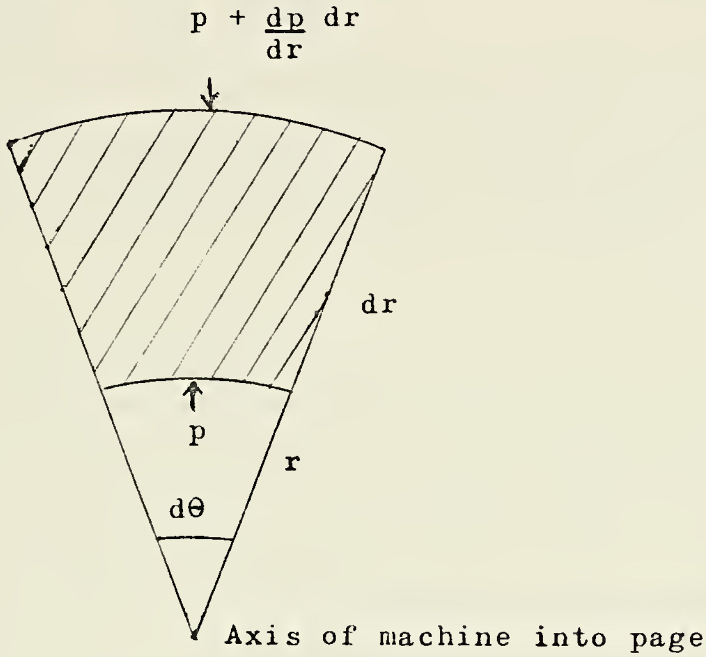


Figure 3

Summuation of the forces acting on this segment gives the equation for this condition of radial equilibrium.

$$\frac{1}{\rho_f}\frac{dp}{dr} = V_\theta^{\,2}/g_o r$$

Using the Gibbs equation and the definition of total enthalpy for an adiabatic condition, the radial equilibrium

equation can be rewritten as

$$\frac{dh_o}{dr} - \frac{Tds}{dr} = \frac{1}{2g_o}\frac{dV_x^{\,2}}{dr} + \frac{1}{2g_o r^2}\frac{d(r^2 V_\Theta^{\,2})}{dr}.$$

Making the assumption that the enthalpy drop is uniformly distributed over the blade radially, and that entropy is not a function of radius, the result is

$$\frac{dV_x^{\,2}}{dr} = \frac{-1}{r^2}\frac{d(r^2 V_\Theta^{\,2})}{dr}.$$

At this point one of several distributions for V_x or V_Θ may be chosen; however, having made the assumption previously that axial velocity is constant throughout the machine, the above reduces to

$$r\ V_\Theta = \text{constant}.$$

This constant may be calculated from the mean conditions and, using the above relationship, the tangential velocities at hub and tip can be calculated. The blade speed is easily calculated at hub and tip as shown previously; and, using trignometric relationships, the complete velocity triangles for rotor and stator are easily calculated in the manner illustrated in section 2.3.3.

The choice of aspect ratio (blade height/blade chord) must now be made. As this is a secondary variable, section 2.3.3, a means must be derived for a logical decision. A study of data presented by Forrester (5) shows that the secondary losses attributed to aspect ratio effects decrease with an increasing aspect ratio; thus a large aspect ratio

would be advantageous from an efficiency standpoint. On the other hand, Horlock (6) states that the bending stresses on the blade may place a limit on the blade chord length and thus limit the feasible aspect ratio. Any aspect ratio chosen should therefore be as large as possible without exceeding a predetermined stress limit. Carmichael recommends a limit of one ton per square inch which was used in the following derivation.

In order to simplify the calculations necessary, the fluid was modeled as incompressible and the blade shape was simplified so that the resulting criteria could be applied to the general case. The maximum stress was assumed to occur in the rotor at the root of the blade and located on the

Blade Root Stresses

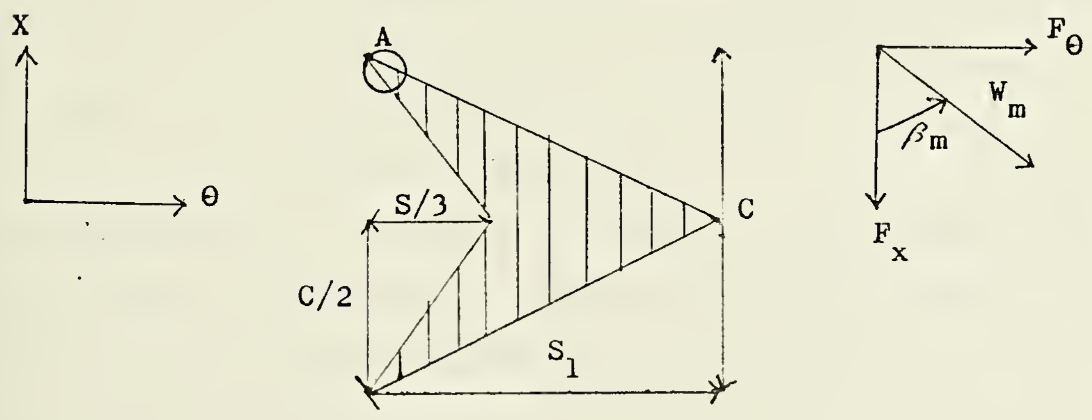


Figure 4

upstream side. By considering the change in tangential momentum across the blade, the bending moment in the tangential direction (Figure 4) can be shown to be

$$M_\theta = \int_{r_h}^{r_t} \rho_f \frac{V_x{}^2}{z} r(r - r_h)(V_\theta)\, dr$$

where z = number of blades per stage.

Integrating this equation, and substituting the Euler work equation and the definition of blade speed gives

$$M_\theta = \frac{\dot{m}\Delta h_o J(r_t - r_h)^2 60\ S_1}{(r_t{}^2 - r_h{}^2)\ (2\pi)^2\ RPM\ r_h\ n}.$$

Simplifying and substituting in the constants yields

$$M_\theta = 591.209\ \dot{m}\Delta h_o (r_t - r_h)\ S_1/RPM\ r_m\ r_h\ n.$$

Since $\tan(\beta_m) = \text{Forces}_{\theta\ \text{direction}} / \text{Forces}_{x\ \text{direction}}$ the bending moment in the axial direction can be described by

$$M_x = M_\theta \tan(\beta_m).$$

For a particular design let K1 be a constant representing

$$K1 = \dot{m}\Delta h_o\ (r_t - r_h)\ 591.209/RPM\ r_h\ r_m\ n$$

then

$$M_\theta = K1\ S_1$$

$$M_x = K1\ S_1 \tan(\beta_m).$$

For the blade shape shown in Figure 4, the moments of inertia about the respective neutral axes can be shown to be

$$I_{\bar{\theta}} = c\ S_1{}^3/69.429$$

$$I_{\bar{x}} = S_1\ c^3/72\ .$$

For stress at the point A,

$$\theta/I_{\bar{x}} = 30.86/c\ S_1^2$$

$$x/I_{\bar{\theta}} = 36/S_1 c^2.$$

The bending stress is then

$$\sigma_b = K1(\mathrm{Tan}(\beta_m)36/c^2 + 30.86/c\ S_1).$$

Using Zweifel's criteria to relate S and c (4),

$$0.8 = 2\ \mathrm{Cos}^2(\beta_{2h})(\mathrm{Tan}(\beta_{1h}) + \mathrm{Tan}(\beta_{2h}))\ S_1/c$$

substituting in the bending stress equation and solving for chord, using the one ton per square inch stress limit mentioned previously,

$$c = (K2\ f)^{\frac{1}{2}}$$

where $K2 = K1\ 32.174/(2000)(144)$

$$f = 36\ \mathrm{Tan}(\beta_{1m}) + 77.15\ \mathrm{Cos}^2(\beta_{2h})\ x$$

$$(\mathrm{Tan}(\beta_{1h}) + \mathrm{Tan}(\beta_{2h})).$$

This approximation for chord is based on a stress limit caused by gas bending forces; however, to account for the fact that other design conditions may impose a lower limit than this criteria, a maximum limit to aspect ratio of 10 will be imposed.

Zweifel's criteria can now be used to determine spacing for the rotor from this calculated chord. Assuming the same chord for the stator, spacing can be calculated there also. As shown in Figure 5, the throat opening can be figured for both the rotor and stator.

Throat Opening

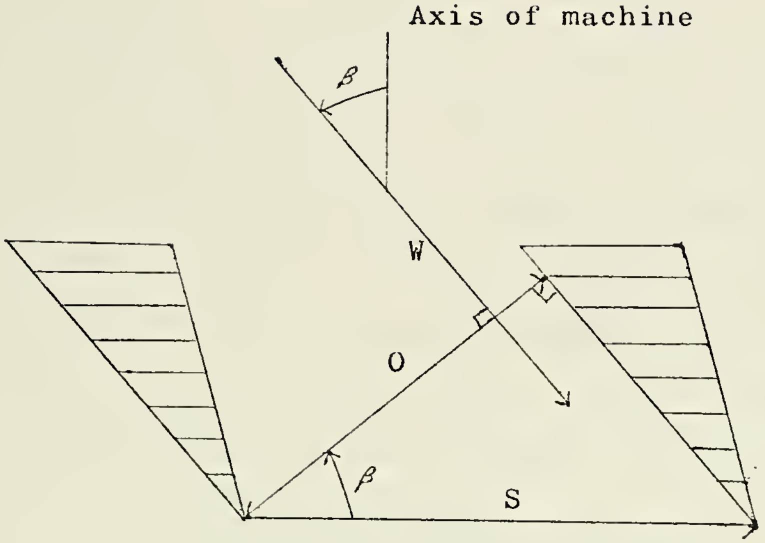


Figure 5

$$O_1 = S_1 \cos(\beta_{1m})$$

$$O_2 = S_2 \cos(\beta_{2m})$$

Knowing the spacing, the number of blades for rotor or stator is simply

$$Z = 2\pi r_h / S.$$

Using the hydraulic diameter of the throat opening, again for both rotor and stator,

$$D = 2\ Ot/(O + t)$$

the Reynolds number for both the rotor and stator can be

calculated as

$$Re_2 = D_2 \, W_{m2}/\nu$$

$$Re_1 = D_1 \, V_{m1}/\nu .$$

2.3.6 Efficiency

The determination of efficiency for axial turbines is a much-debated subject. Craig & Cox (3), Smith (5), and Ainley and Matheson (5), all correlate their data using the work and flow coefficients mentioned previously; while Soderberg (4) correlates using gas deflection angle. In either case the primary loss is related to blade profile loss. Secondary losses are attributed to many factors, some of them being:

1) Clearance loss
2) Reynolds number effects
3) Aspect ratio
4) Guide loss
5) Runner loss
6) Guide gland loss
7) Lacing wire loss
8) Balance hole loss
9) Wetness loss
10) Disc windage loss
11) Partial admission loss
12) Annulus wall loss
13) Cavity loss.

While including all of these factors would give a most accurate efficiency prediction, the accuracy required in a preliminary design doesn't require the inclusion of them all.

Soderberg's correlation was chosen in that it provides accuracy consistent with the intent of the program; yet it is simple enough to be easily programmed on a computer. This correlation takes into account profile loss, clearance loss,

aspect ratio, and a Reynolds number effect. These losses are equated to a fraction of the kinetic energy available to the turbine and a total to total efficiency calculated.

The method consists of finding a primary loss factor, based on gas deflection angle for both stator and rotor. This loss factor is modified for aspect ratio and Reynolds number effects, and then an efficiency calculated. The efficiency is then modified by a factor which takes into account the clearance losses.

$$\zeta_o = f \text{ (Gas Deflection)}$$

$$\zeta_1 = ((1+\zeta_o)(.975+0.075\ c/t))-1$$

$$\zeta_2 = \zeta_1 \left(\frac{10^5}{Re}\right)^{.25}$$

The above are calculated separately for both stator and rotor. The efficiency is then:

$$\eta_1 = \left(1+\frac{\zeta_2 \text{Stator } V_1^2 + \zeta_2 \text{Rotor } W_2^2}{2U(V_{\theta 1m})}\right)^{-1}$$

$$\eta_{tt}(\text{Stage}) = \eta_1 \left(\frac{\text{Area Annulus - Tip Clearance Area}}{\text{Area Annulus}}\right).$$

The primary loss coefficient is one which can be programmed easily in a curve-fitting subprogram. The tip clearance area was calculated based on a tip clearance of one percent of the tip radius. This gave a constant factor of 0.99. Assuming a linear distribution of efficiencies between the first and last stages means that the machine efficiency is the average efficiency.

$$\eta_{Machine} = (\eta_{First\ Stage} + \eta_{Last\ Stage})/2.$$

2.3.7 Miscellaneous Machine Parameters

In order to more easily picture the machine size, all pertinent radii are converted to diameters.

A machine unit length is calculated using the average stage chord and the number of stages.

The maximum blade centrifugal stress occurs in the last stage at the rotor blade root. This can be written as follows:

$$\sigma_c = \frac{\rho_{mat.}}{g_o A_h} \int_{r_h}^{r_t} \omega^2 r\, A_h\, dr.$$

If the blade is assumed to have no taper, A_h is constant. Substituting in the constants and assuming the density of steel is approximately 0.29 lbm/in^3 gives

$$\sigma_c = 0.001186\ RPM^2\ r_m\ t.$$

2.3.8 Turbine Limitations

Certain limitations must be placed on the mathematical model previously described, so that it won't produce turbine parameters for a machine that is either physically impossible or not consistent with the assumptions of the model. These limitations will be in the form of numerical checks in the program and will cause the calculations to be terminated if they are not satisfied.

The first check is for excessive gas deflection. While

this in itself is not an impossible situation, only leading to a lower efficiency, the data available for the Soderberg correlation only accommodates a gas deflection of up to 140°. Since the efficiency decreases rapidly at angles greater than this, no harm is done in limiting gas deflection to 140°.

Reaction is another parameter which is limited. The upper bound has been discussed earlier in section 2.3.3. A check must be made, however, to insure that at no point does reaction become negative. Physically, a negative reaction would mean that within a given stage, at some point, a pressure rise was occuring to be compensated for by a larger pressure drop at another point. The overall condition, of course, would be a pressure drop. In order to avoid this condition which brings about higher losses, a check should be made to insure non-negative reaction. Negative reaction can only occur with large V_θ and since it is assumed that $r\ V_\theta =$ constant, the check should be made at the smallest radius, the last stage hub.

In solving for a mean radius, in section 2.3.4, no continuity check was made. Without a check for continuity it would be possible, using the model described, to compute a negative machine diameter. In order to avoid this and to allow for bearings, shafting, and blade attachment, a check will be made to insure that the hub radius calculated is greater than 50 percent of the tip radius.

The one dimensional compressible flow equations used throughout also pose another limitation. The fluid flow throughout the machine is assumed to be subsonic, and the design has proceeded such that no consideration has been given to the shock effects caused by supersonic flow. To insure subsonic flow, a check will be made on the two largest velocities to insure a Mach number less than one. The velocities are the last stage hub velocity at rotor inlet and the last stage tip relative velocity at rotor outlet.

In addition to these above limitations, there are physical limitations imposed by the computer programming itself. These are listed in the appropriate appendices.

2.4 Radial Turbine

2.4.1 General

The term radial turbine is used to describe a wide range of turbine configurations. In the general sense, it describes a turbomachine where the fluid enters the rotor with no appreciable axial component of velocity. Whether the fluid flows radially inward or radially outward and whether it leaves the rotor in a radial or axial direction depends upon the context of the discussion. The choice made here to limit the radial design to that of only one configuration was based on practicality. The 90 degree inward flow radial turbine, in which the fluid enters the rotor in a radial but leaves in an axial direction, is the most common type of radial turbine (7). It

has the advantage of the strongest structural strength, also a factor which can be important at high rotational speeds. Only single stage turbines will be considered as these are the most common also.

The methodology of designing the radial turbine can be summarized in the following four steps: rotor design, nozzle design, scroll design and efficiency correlation. The station designations are as shown in Figure 6.

2.4.2 Output Required

The parameters necessary to describe the turbine of Figure 6 can be narrowed down to the following list for preliminary design:

1) Nozzle Inlet and Outlet diameters
2) Nozzle blade height
3) Rotor Inlet diameter
4) Rotor Outlet diameters at hub and tip
5) Maximum Scroll Area
6) Unit length
7) Specific speed
8) Efficiency
9) Velocity triangles at
   A) Nozzle Inlet
   B) Rotor Inlet and Outlet.

The velocity triangles for rotor outlet will be described at hub, mean, and tip locations.

2.4.3 Input

The parameters necessary to describe a radial machine can be categorized in the same manner as the axial parameters (section 2.3.3): those which describe the mass flow rate, those which describe the work accomplished per stage, and those which describe the performance of a stage.

Radial Turbine Station Designations

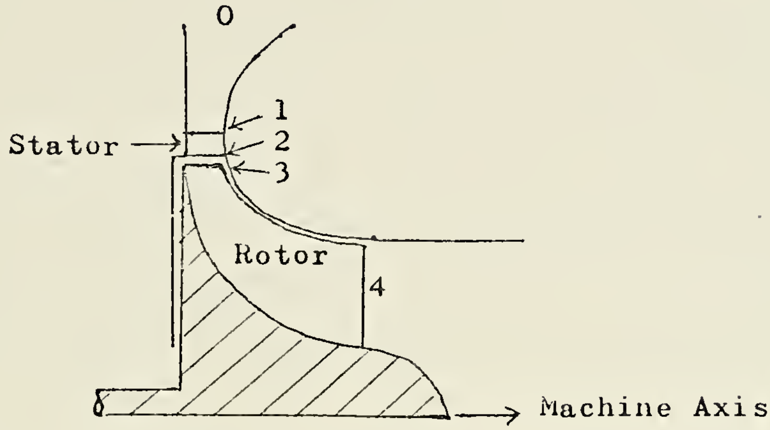


Figure 6

As it has been previously assumed, that the radial turbine design is for a single stage machine, the work accomplished per stage is the same as the total work to be accomplished by the cycle parameters, section 2.2.3.

The mass flow rate for this machine can also be described in a manner similar to that for the axial machine and which is presented in section 2.2.2. The variables necessary for this representation are the mass flow rate and the number of machines operating in parallel along the fluid flow path.

Performance estimation is more standardized for radial turbines than axial turbines. Through the use of a non-dimensional specific speed parameter it is possible to correlate

efficiency with rotational speed, volumetric flow rate, and work accomplished, for turbines designed with the same basic assumptions and geometric relationships. (8) There are several specific speed parameters in use which will be discussed in a later section; however, for the immediate purpose, rotative speed is a necessary input. One last input remains and that relates to an assumption to be made in the succeeding section. Without explaining this assumption in detail, it is sufficient to say that the number of radial blades of the rotor should be included as an input.

The input for the radial design can be summarized as follows:

1) Mass flow rate of the cycle
2) Number of units in parallel
3) Shaft revolutions per minute
4) Number of radial rotor blades
5) Cycle conditions of section 2.2.

2.4.4 Rotor Calculations

The first step in determining the rotor parameters is to determine the conditions at rotor inlet (Station 3). Borrowing an analysis from the field of centrifugal compressor design, it can be shown that the optimum performance is obtained when the following relationships hold at rotor inlet. (8)

$$\xi = 1 - \frac{2.0}{Z}$$

$$\frac{V_{\theta 3}}{U_3} = \xi$$

where Z is the number of radial rotor blades.

This factor (ξ) is often called the slip factor. If the velocity at rotor exit is assumed to be entirely in the axial direction for reasons similar to those given for the axial turbine in section 2.3.4, then the slip factor may be used in the Euler equation to solve for the radius at rotor inlet.

$$\Delta h_o = \frac{\xi U_3^2}{g_o J}$$

where $U_3 = \pi \, RPM \; r_3/30$.

Substituting the constants into this gives:

$$r_3 = \frac{1510.8233}{RPM} \frac{(\Delta h_o)^{\frac{1}{2}}}{(\xi)^{\frac{1}{2}}}.$$

The condition necessary to avoid negative velocities at rotor inlet (2) is:

$$\frac{V_{m3}}{U_3} \geq \frac{2\pi}{Z}.$$

Combining this expression with that of the slip factor, it is possible to determine the optimum inlet flow angle, α_3.

$$\mathrm{Tan}\,\alpha_3 = \frac{V_{\theta 3}}{V_{m3}} = \frac{V_{\theta 3}/U_3}{V_{m3}/U_3}$$

$$\alpha_3 = \mathrm{ArcTan}\ \frac{(Z-2)}{(2\pi)}.$$

It is now possible to determine the complete velocity triangles at rotor inlet, Figure 7.

Using the above relationships and the definition for U_3,

$$V_{m3} = \frac{2\pi}{Z} U_3$$

$$U_3 = \frac{\pi\, RPM}{30}\, r_3$$

$$V_{m3} = \frac{994.0819}{Z} \frac{(\Delta h_o)^{\frac{1}{2}}}{(\zeta)^{\frac{1}{2}}}$$

$$V_3 = V_{m3}/\cos(\alpha_3)$$

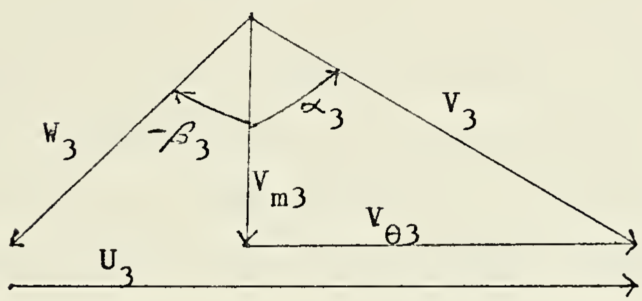
$$V_{\theta 3} = V_3 \sin(\alpha_3)$$

$$\beta_3 = \mathrm{ArcTan}(V_{\theta 3} - U_3)/V_{m3})$$

$$W_3 = V_{m3}/\cos(\beta_3).$$

Radial Turbine Velocity Triangles

Rotor Inlet



Rotor Outlet

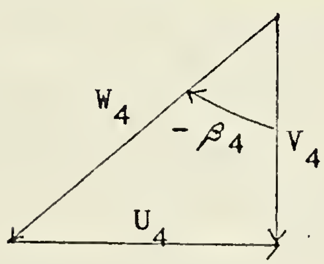


Figure 7

Using the compressible flow relationships, the flow area at inlet is obtained. The flow area and annulus area are again related by the cosine of the inlet flow angle as shown in Figure 2. The annulus area in this case, however, consists of the circumference times the blade height. From this the blade height can be calculated.

At rotor outlet, the assumption is made, as stated previously, that the tangential velocity is zero. Watanabe (9) states that there is a definite ratio of the mean outlet radius to inlet radius which will give the peak efficiency. This ratio is approximately 0.6. It has also been somewhat traditional to design for a relative outlet angle at the shroud of minus 60°. (2) This represents the compromise between a low RPM machine and a low loss machine. Applying both of these criteria in a modified way, an outlet velocity triangle at mean, with a relative flow angle of minus 60° was chosen. This later proved to correlate well with previous work as shown in section 3.6 and avoided an iterative procedure for calculating rotor outlet conditions.

Using these assumptions, the outlet velocity at mean, in terms of input variables, is

$$V_{m4} = 9.1347(\Delta h_o/\eta)^{\frac{1}{2}} \text{ (Radius Ratio)}$$

The value of 0.6 specified above is used as a first estimate for radius ratio. If the resulting outlet conditions do not meet the checks specified in section 2.4.9, then the ratio is

increased and outlet conditions recalculated.

As β_{4m} is minus 60°, the other outlet conditions at mean are

$$U_{4m} = \frac{\pi \, RPM}{60} \, r_{4m}$$

$$W_{4m} = V_{4m}/\text{Cos } (\beta_{4m}).$$

The compressible flow relations are again used to determine the outlet flow area. In this case, since the outlet velocity is in only an axial direction, the flow area is equal to the annulus area. The outlet height may be calculated by the same method used in section 2.3.5 to calculate axial blade height:

$$\text{outlet height} = \text{Area ann}/2\pi r_{4m}.$$

Making the assumption that the outlet velocity is constant with respect to radius, and knowing the outlet hub and tip radii, the velocity triangles for these points are calculated as previously shown, with the exception that the relative outlet angle varies with the radius, or more simply

$$\beta_4 = \text{ATAN } (-U_4/V_{4m}).$$

2.4.5 Stator Calculations

The stator calculations are relatively simple. Watanabe (9) states that the clearance between stator outlet and rotor inlet should be twice the passage height. If it is assumed that the blade or passage height is the same for both rotor and stator, then the stator outlet radius is simply the radius

at rotor inlet plus two times the blade height. As in most cases this gap will not be large, and as it can be assumed that the moment of momentum remains constant across the gap,

$$\frac{r_2}{r_3} = \frac{V_{\theta 3}}{V_{\theta 2}}$$

where $r_2/r_3 \approx 1$.

This means that the velocity triangle at stator outlet is substantially the same as at rotor inlet and will not be recalculated.

The radial spacing between blades or pitch, assuming that there are as many stator blades as rotor blades, is the circumference divided by the number of blades. The throat opening can also be determined in a manner similiar to that of section 2.3.5. Using a pitch to chord ratio of 0.6, the blade chord may be calculated. Using the assumption that $\cos \alpha_2$ = opening/spacing, the approximate radius at the stator inlet can be calculated as shown in Figure 8.

$$\text{Angle } 1 = \frac{\pi}{2} - \frac{\pi}{Z}$$

$$\text{Angle } 2 = \frac{\pi}{2} - \alpha$$

$$\text{Angle } 3 = \frac{\pi}{Z} + \alpha_3$$

$$\text{Side } 1 = r_2 + C \cos\left(\frac{\pi}{Z} + \alpha_3\right)$$

$$\text{Side } 2 = C \sin\left(\frac{\pi}{Z} + \alpha_3\right)$$

$$\text{radius}_1 = (\text{Side } 1^2 + \text{Side } 2^2)^{\frac{1}{2}}$$

Radial Turbine Stator Configuration

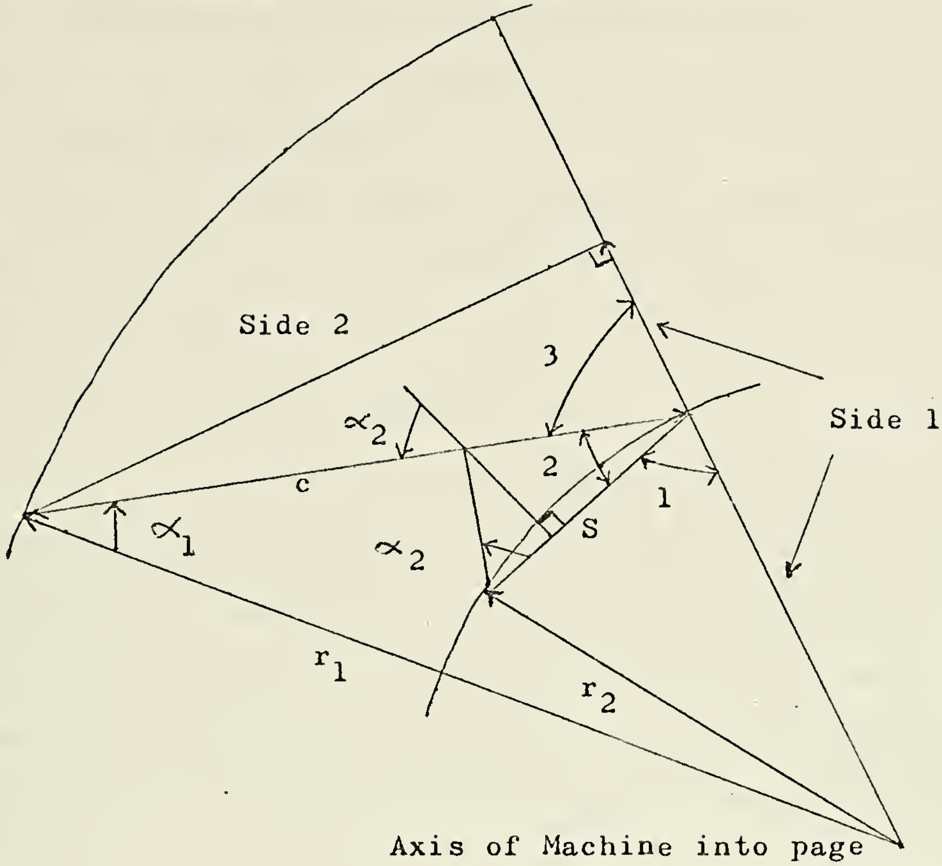


Figure 8

The inlet angle can now be determined using the law of Sines:

$$\sin(\alpha_1) = \frac{r_2}{r_1} \sin\left(\pi\left(1-\frac{1}{Z}\right)-\alpha_2\right).$$

For the passage from stator inlet to outlet, it can be assumed that the fluid is incompressible. This is a valid assumption as the Mach number is low at rotor inlet. Since the mass flow rate must remain constant, the velocity change is proportional to the flow area change. This can be expressed as

$$V_1 = V_3 \frac{r_3}{r_1}.$$

Having determined the inlet angle, α, the rest of the velocity triangle is also easily obtainable.

2.4.6 Scroll Calculations

The scroll shape assumed is shown in Figure 9. It is assumed that the area is distributed radially in a linear relationship, thus:

$$\text{Area}_\theta = A_{max} \left(1 - \frac{\theta}{2\pi}\right)$$

where θ = radial angle in radians.

The maximum area needed can be approximated if it is assumed that the tangential and meridional components of velocity do not vary as a function of the radial angle. The maximum area is then equal to the flow area at stator inlet which in turn is related by the cosine of the inlet angle.

Radial Turbine Scroll Configuration

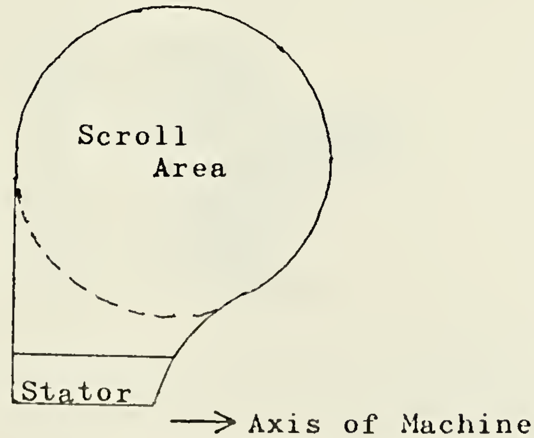


Figure 9

$$A_{max} = 2\, r_1 \pi t \cos(\alpha_1)$$

where t is the blade height at rotor inlet

2.4.7 Radial Efficiency

As with the axial turbine, the theoretical determination of efficiency for a radial turbine is a much debated subject. Wood (4) and others have correlated efficiency with the non-dimensional specific speed mentioned previously, section 2.4.3; however, this does not give an exact enough answer for the purposes here. A more analytical method is needed.

Heitt & Johnston (10) relate the performance of several specific turbines to several design variables; however, the method is rather a narrow one intended to experimentally evaluate performance as a function of specific variables, rather than provide an analytic efficiency determination. Rodgers (11) proposes a method of determining efficiency which accounts for losses caused by:

1) Disc friction
2) Nozzle shape
3) Rotor friction
4) Rotor blade loading
5) Rotor secondary flow
6) Rotor incidence
7) Rotor clearance
8) Rotor cooling
9) Exhaust diffuser.

This method, while verified to within 2 percent, takes into account considerably more variables than are necessary for the purposes of this mathematical model.

The method chosen was that used by Rohlik (8) in his

study of the effects of turbine geometry on efficiency. This method is a compilation of several experimental correlations relating the efficiency to:

1) Stator and boundary layer losses
2) Clearance losses
3) Rotor windage losses
4) Rotor boundary layer losses
5) Exit velocity losses

This method provided the needed precision and was based on the work of many acknowledged experts in the field; Baljé (12), Heitt & Johnston (11), Wood (7), Stewart, Whitney & Wang (13) and Shepherd (14) to name a few. The correction to account for exit velocity, however, is omitted; thus the efficiency calculated is a total to total efficiency and is compatible with the efficiency calculated for the axial turbine. If a total to static efficiency is desired, it can be easily calculated from the data given for the outlet velocity triangles as:

$$\eta_{ts} = \frac{\eta_{tt}}{\eta_{tt} + \dfrac{V_{4m}^{2}}{2g_o\, J(h_{o1}-h_{o2s})}}$$

where η_{tt} = efficiency (total to total)

η_{ts} = efficiency (total to static)

This conversion factor can also be applied to determine total to static efficiency for the axial turbine by using V_{2m} in lieu of V_{4m}.

The mechanics of the method consist of relating each loss to an associated enthalpy change and then summing these in a fractional way to determine efficiency. The first loss considered is that of rotor windage. Rotor inlet density is required and also an absolute viscosity (μ), at rotor inlet. The density is computed using the one-dimensional compressible flow relationships.

$$\rho = \frac{\rho_o}{\left(1 + \frac{\gamma-1}{2} M^2\right)^{1/(\gamma-1)}}$$

$$\rho_o = P_o/RT_o$$

$$\rho = \frac{P_o}{RT_o} \left(1 - \frac{\gamma-1}{2} M^2\right)^{1/(\gamma-1)} \Big/ \left(1 - \left(\frac{\gamma-1}{2}\right)^2 M^4\right)^{1/(\gamma-1)}.$$

For Mach numbers less than one, the denominator approaches one and the density is then:

$$\rho = \frac{P_o}{RT_o} \left(1 - \frac{\gamma-1}{2} M^2\right)^{1/(\gamma-1)}.$$

Using this density, the absolute viscosity, (μ) can be calculated using the kinematic viscosity (ν) given as an input.

$$\mu = \rho \nu.$$

A Reynolds number for the rotor inlet is defined as:

$$R_e = \frac{2U_3 r_3}{\nu_3}.$$

Substituting in the constants and the definition of rotative velocity:

$$R_e = 0.2094395 \text{ RPM } r_3^{\,2}/\nu_3.$$

The windage loss can now be computed as:

$$L_w = \frac{0.56\rho_1\mu_1 D_1^{\,2}}{R_e^{\,.2}\ \dot{m}_i\ 10^6}$$

or

$$L_w = \frac{2.24\rho_3^{\,2}\nu_3 r_3}{R_e^{\,.2}\ \dot{m}_i\ 10^6}.$$

The clearance loss is expressed as:

$$L_c = h_o \mathcal{N}$$

where $\mathcal{N}$ is defined as

$$\mathcal{N} = \tfrac{1}{2}\left(\frac{.004r_3}{t_3} + \frac{.005r_{4t}}{t_4}\right).$$

The rotor boundary layer loss equations involve several assumptions (8) which go beyond the scope of this model; however, they are consistent with good practice and those made previously in this model. It is assumed that the resulting equations apply to this model. This holds true for the stator boundary layer loss equations also.

The average of inlet and outlet blade spacing is defined as:

$$S = \frac{\pi}{Z}(r_3 + r_{4h} + r_{4t}).$$

The solidity is defined as:

$$\sigma = \frac{1.6r_{4m}}{S}\left((r_{4m}/r_3)-1\right).$$

A loss coefficient (e_r) is calculated by:

$$e_r = \frac{(0.017\sigma)}{(0.5-0.003Z-0.017\sigma)} \left(1+\frac{1.9S}{(t_3+t_4)}\right).$$

and the total boundary layer loss is given by:

$$L_r = \frac{e_r}{1-e_r} \; \frac{W_{4m}^{2}}{2g_o J} .$$

In a like manner for the rotor, a stagger angle is defined as:

$$\alpha_{st} = (\alpha^1+\alpha_3)/2$$

where α^1 is given by

$$\alpha^1 = \mathrm{ArcTan}\left(\frac{\mathrm{Sin}\alpha_3}{\frac{2t_3}{r_3} + \mathrm{Cos}\alpha_3}\right).$$

The loss coefficient (e_s) is calculated from

$$e_s = \frac{0.0076}{\mathrm{Cos}\alpha_3-0.025} \left(\frac{1+\mathrm{Cos}\,\alpha_{st}}{0.7}\right),$$

and a total loss for the stator boundary layer calculated from

$$L_s = \frac{e_s}{1-e_s} \left(\frac{V_3^{2}}{2g_o J}\right).$$

The efficiency is found from

$$\eta_{tt} = \Delta h_o-L_w-L_c/(\Delta h_o+L_s+L_r).$$

2.4.8 Miscellaneous Parameters

Specific speed seems to be defined differently by each researcher. Some definitions are non-dimensional and others

are not. Wood (7) and Rohlik (8) define their specific speed similiar to that used in centrifugal pump theory as

$$N_s = \frac{RPM(Q_4)^{\frac{1}{2}}}{H^{3/4}}$$

where Q_4 is the volumetric flow/sec

H is the ideal head drop across the turbine in feet.

This, however, is dimensional and for that reason the non-dimensional specific speed recommended by Carmichael (2) is used instead.

This is defined as

$$N_s = \frac{RPM}{60} \frac{Q_4^{\frac{1}{2}}}{(g_o \; J\Delta h_o)^{3/4}}.$$

The rotor outlet volumetric flow can be calculated as

$$Q_4 = \dot{m}_i / \rho_4$$

where ρ_4 is obtained using the compressible flow relationship of the previous section.

As a matter of interest the specific speed of Wood et al may be obtained from that used in the model by using the following formula:

$$N_s \text{ (Wood)} = 810.04455 \; N_s \text{ (Carmichael)}.$$

The axial unit length of the rotor is a parameter which must be calculated in detail for the final design; however, in order to simplify the calculations in the mathematical model a simpler method is used. Based on the data given in several

of the references, on the average, the unit axial length is on the order of the inlet radius less the outlet hub radius, thus for the purposes here,

$$\text{length} = r_3 - r_{4h}.$$

The machine diameters given as output are simply calculated from the previously derived radii.

2.4.9 Radial Limitations

The specific speed parameter has an important influence on the shape of the turbine. In order that the model not generate dimensions which were unacceptable from a geometric point of view (i.e. a radical unconventional design in which other factors such as shroud clearance might have an unaccounted for effect), the input parameters are first checked to determine the specific speed of the proposed turbine. If this specific speed is below 0.05 or above 0.25 the input parameters will not produce a design which is conventionally feasible.

The calculation of stress in the rotor is a complicated subject and one which is beyond the scope of the preliminary design phase. Wood (7), however, provides a means of determining whether a design will be stress limited. Specific speed is written in terms of outlet area, RPM, rotor tip speed and several fluid velocities. Maximum expected values for a stress limited case are then used to solve for a specific speed. This specific speed was approximately twice that

which would be allowed by the previous criteria and, therefore, the assumption is made that the radial turbines allowed by this model will not be stress limited.

As in the axial turbine a continuity check must be made at rotor exit to assure that negative diameters are not calculated. The rotor outlet radius is checked to insure that it is at least 10 percent of the inlet radius. This differs from the previous 50 percent used in the axial turbine; however, the radial rotor doesn't have to incorporate a through shaft and thus can be of a much smaller inner diameter. If the radius in question is too small, the radius ratio used in calculating V_{m4}, section 2.4.4, is increased by 2 percent. This has the effect of increasing the density of the fluid and thus decreasing the outlet annulus area required.

The calculations described here for the radial turbine do not account for any shock effects caused by operation at a Mach number greater than 1. Even without this correlation, however, Rodgers (11) states that Mach number effects are not important until M = 1.2. Carmichael (2) recommends a limit of 1.1. The lower limit was used to be consistent with other assumptions made previously. The two velocities likely to be the largest, the outlet tip relative velocity, and the rotor inlet velocity are checked to insure that they are below a Mach number of 1.1. If not, the input parameters result in an unfeasible turbine.

EVALUATION OF THE MODEL

3.1 General

In order to facilitate the preliminary design of turbines using the mathematical model developed in the previous chapter, it was necessary first to program the model, and then to run the program over a wide range of inputs in order to verify the results with existing correlations; or in the case of the simpler geometric relationships, with hand calculations. The areas which were examined for validity and the methods used to establish same can be summarized as:

Area	Method
Simple mathematical relationships	hand calculations
Aspect ratio selection	hand calculations of existing turbine
Soderberg correlation function	comparison with original function
Axial turbine efficiency	comparison with other published correlations
Radial turbine efficiency	comparison with other published correlations

Each of these areas will be described in more detail in the following sections.

The cycle chosen for the validation was the Zener sea cycle as proposed by Clarence Zener (15). This cycle is similiar to others (16) which seek to take advantage of the temperature differences existing in the world's oceans, and is representative of the type of cycle for which the mathe-

matical model was designed to be used to evaluate. The details of this cycle and of the individual parameters which were examined are contained in Appendix F. It is sufficient to say here that over 50 different combinations of input parameters were examined for each of two different working fluids. This range of input variables provided a representative picture of the design surface, and allowed the results to be broad enough to correlate with existing data over a wide range.

3.2 Simple Mathematical Relationships

This facet of the model is the predominent one, as explained in section 1.3, and also the easiest to verify. The major portion of the model is based on traditional theory which has been well proven in the laboratory. It is not the purpose here to attempt to revalidate the conclusions of researchers far more qualified than the author. The emphasis has been placed instead on choosing accepted relationships and correlating them into a usable model.

Based on this, the validation of this segment consisted of hand calculating the various mathematical relationships and verifying the results of the computer program. To this end, the model has been debugged of all forseeable mechanical type mathematical errors.

3.3 Aspect Ratio Correlation

The decision to develop an internal choice of aspect

ratio based on blade bending stress for the axial turbine, rather than have the designer predetermine it, was based on primary and secondary input factors as explained in section 2.3.3. It was necessary then to verify that the internal choice was a reasonable one, consistent with existing designs.

This was carried out by applying the correlation developed in section 2.3.4 to an existing design for an intermediate stage turbine, developed for marine propulsion use and described by T. B. Hutchinson in reference 17. This design was for an 8 stage steam turbine operating at 5036 RPM and producing 8940 horsepower at 90 percent efficiency with an inlet temperature and pressure of 1000^{o}F and 225 psig respectively. Using the data supplied by Hutchinson in reference 17, and making assumptions, where necessary, of input variables which would optimize efficiency, the correlation of section 2.3.4 was applied to the design and an aspect ratio was computed. This aspect ratio was approximately 1.4. The aspect ratio chosen in the actual design was approximately 1.5.

While this simple point verification does not guarantee validity over the complete range of input variables, it does show that the relationship is reasonable. When this relationship is combined with an upper limit for aspect ratio, to take into account the fact that other factors than stress may limit the design, the resulting range of variation of aspect ratio (0.25 to 10.0) is consistent with existing designs (5) and is

assumed to provide a reasonable result.

3.4 Soderberg Correlation

In determining efficiency for the axial turbine, section 2.3.5, it was necessary to formulate a function which approximated the curve of emperical data given by Soderberg (4) for the correlation of gas deflection angle and the primary loss factor. This was accomplished in a computer subprogram through the use of LaGrange polynomials to curve fit the given data to a third order approximation of the function (18). Without explaining the details of the theory of curve fitting, it is sufficient to state that within a given subinterval the correlation was represented by the function

$$W = X_1 Y_1 + X_2 Y_2 + X_3 Y_3$$

where X_1, X_2 and X_3 are third order functions dependent upon the value of the abscissa given, and Y_1, Y_2, and Y_3 are values of the ordinate which bracket the given abscissa value.

The simulation of this function is shown in Figure 10.

3.5 Axial Turbine Efficiency

Although the correlation adopted for determination of axial total to total efficiency has been previously validated, it was necessary to compare the results obtained with the model to other correlations in order to assure that the correlations provided reasonable results for the assumptions made in the model. This was accomplished by comparing the computed efficiency of several example cases for the Zener

Subprogram SODCOR Simulation of Soderberg ζ Parameter

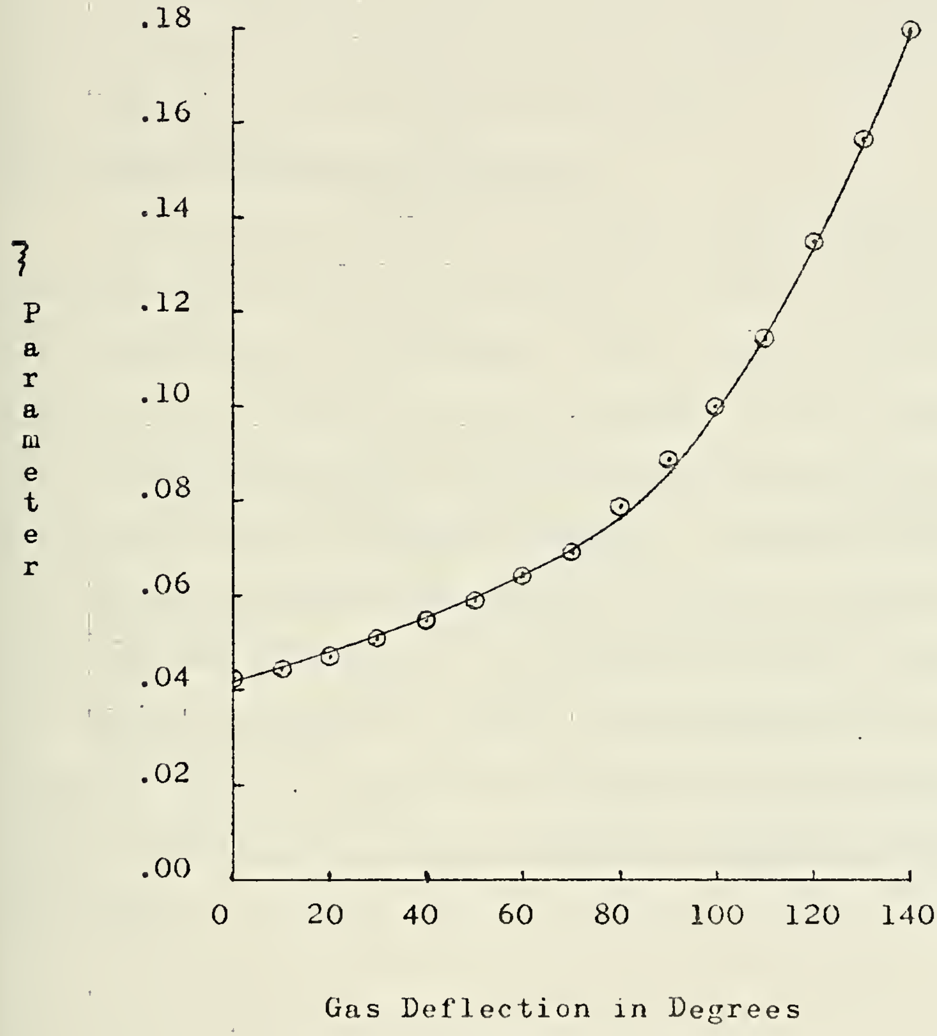


Figure 10

cycle with the correlations presented by Craig and Cox (3). This comparison is shown in Figure 11.

This comparison does not account for the differences between the two correlations in their accounting for secondary loss parameters; however, the results are close enough to show that the application of Soderberg's method is valid and consistent with accepted practices.

3.6 Radial Turbine Efficiency

It was also necessary to compare the computed radial turbine efficiency to other correlations in order to verify its application in the model. This can be more easily accomplished in the radial turbine than the axial through the use of the non-dimensional specific speed parameter mentioned in section 3.4.8. Several examples, covering a wide range of specific speeds, were compared with the correlations of Wood (2) and Rohlik (8). The results of this correlation are shown in Figure 12. The close agreement over the wide range of specific speeds shows that the determination of radial efficiency is also valid and consistent with accepted practice.

Axial Turbine Efficiency Comparison

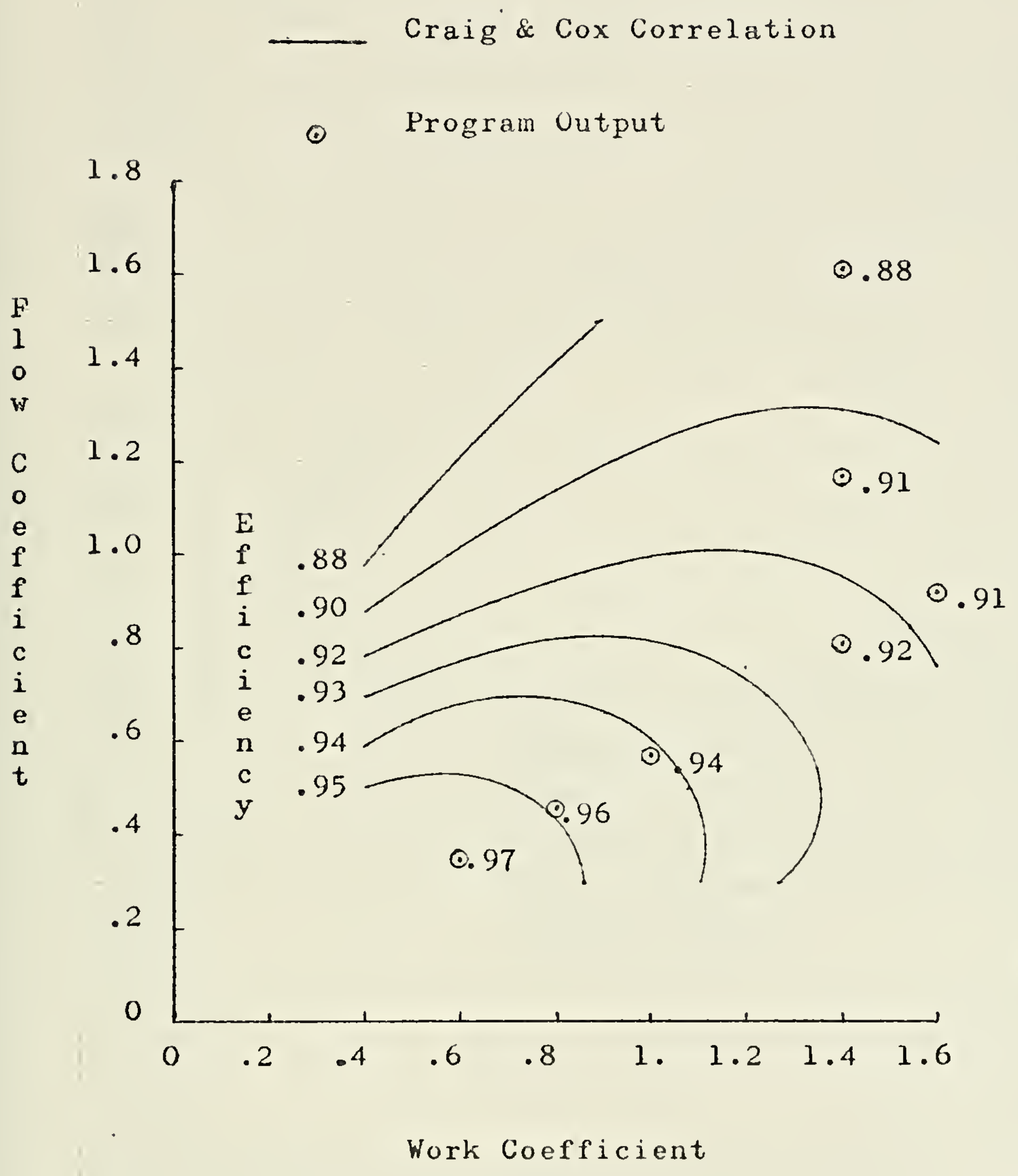


Figure 11

Radial Turbine Efficiency Comparison

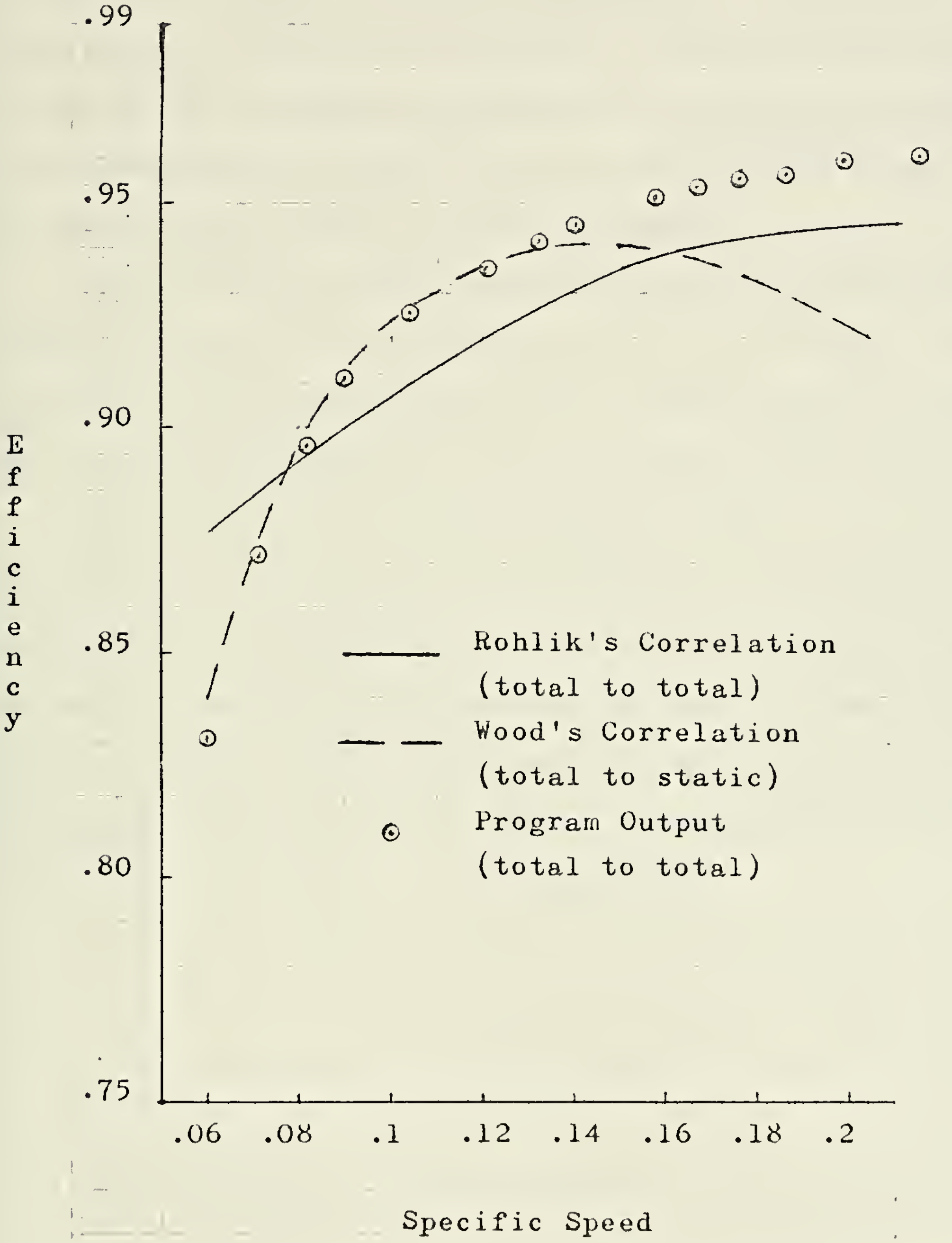


Figure 12

CONCLUSIONS

The formulation of conclusions about a project of this nature is a somewhat limited undertaking. The object of the thesis is to compile existing data, relationships, and methods in the field of turbine design and to formulate and computerize a mathematical model, based on this compilation. This has been accomplished as shown in chapter 2.

The validity of the mathematical model has been established in chapter 3 to the extent that it is valid, consistent with the assumptions stated in its formulation, and within the requirements posed by the preliminary design stage.

The actual use of this programmed model, as explained in Appendix F, has shown that through the utilization of this program, or programs of this type, the designer may explore a wide range of input variables in a comparably short amount of time. This enables him to more fully examine the design spectrum and to develop a qualitatively better estimation of thermal cycle feasibility.

RECOMMENDATIONS

In developing the mathematical model described in chapter 2, it became apparent that there were four areas where the model could be expanded which would increase its usefulness to the designer. The first of these is in the area of the flow relationships utilized. The one dimensional flow relationships do not completely describe the flow. Expansion of these relationships to include shock effects would broaden the applicability of the model.

The second and third areas which could be expanded are integrally related to one another. They entail enlarging the areas of the machine which are described by the model, perhaps including material limits and selection, disc design, bearing and shafting design. Concurrent with this would be the increasing number of input variables necessary to describe the machine. This would be limited by the amount of detail required to satisfy a particular designer's definition of preliminary design.

The last area which was not included in this model is that of cost estimation. Information in this area is generally proprietory and not easily obtainable. Most of the information available is limited in scope and not applicable to the general case as described herein. This area in itself could be the subject of a dissertation, but certainly more work is needed before it is acceptable for inclusion in a model similar to that developed in chapter 2.

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APPENDIX A

A.1 Variable List

The following is a list of variables as they appear in the main program and the subprograms. They are listed in the order in which they appear within the body of the programs. Subscripted variables refer to machine inlet and outlet, subscripts 1 and 2 respectively. The units of these variables are those listed in the Notation section, pages 7 to 9, with the exception of pressure, kinematic viscosity and all angles which are internally converted to lbf/ft^2, ft^2/sec and radians respectively.

MAIN PROGRAM

Cycle and Machine Inputs

H01	inlet stagnation enthalpy
T0	stagnation temperature
P0	stagnation pressure
NU	kinematic viscosity
H02	outlet stagnation enthalpy
H02S	outlet isentropic stagnation enthalpy
T02S	outlet isentropic stagnation temperature
P02S	outlet isentropic stagnation pressure
CP	specific heat at constant pressure
GAMA	ratio of specific heats
CV	specific heat at constant volume

R	universal gas constant
DHO	stagnation enthalpy drop across machine
RJ	universal gas constant times 778
MM	number of combinations of input parameters entered
AXN	number of turbine stages-axial
REAM	reaction at mean
RPMX	shaft revolutions per minute-axial
ALFAMN	rotor inlet true flow angle at mean
UNITX	number of axial turbines in parallel on flow path
RPMR	shaft revolutions per minute-radial
UNITR	number of radial turbines in parallel on flow path
ZR	number of radial blades
WW	total mass flow rate of working fluid

<u>Axial Turbine</u>

RADM	radius at mean
VTMN	tangential velocity component at rotor inlet, mean radius
VMN	true velocity at rotor inlet, mean radius
VX	axial velocity component
WMR	relative velocity at rotor outlet, mean radius
TALFMN	tangent of ALFAMN
BETAMN	relative flow angle at rotor inlet, mean radius
BETAMR	relative flow angle at rotor outlet, mean radius
ZETAN1	primary Soderberg efficiency parameter at rotor inlet

ZETAR1	primary Soderberg efficiency parameter at rotor outlet
CON	product or radius times tangential velocity component
WMN	relative velocity at rotor inlet, mean radius
UMN	tangential rotor velocity at mean radius
FLFN	function f (section 2.2.3)
AFLO	flow area
AANN	annulus area
T	blade height-axial
RADT	tip radius
RADH	hub radius
VTTN	tangential velocity component at rotor inlet, tip radius
VTHN	tangential velocity component at rotor inlet, hub radius
ALFATN	rotor inlet true flow angle at tip radius
ALFAHN	rotor inlet true flow angle at hub radius
VTN	true velocity at rotor inlet, tip radius
VHN	true velocity at rotor inlet, hub radius
UTN	tangential rotor velocity at tip radius
UHN	tangential rotor velocity at hub radius
WTTN	relative tangential velocity component at rotor inlet, tip radius
WTHN	relative tangential velocity component at rotor inlet, hub radius
BETATN	relative flow angle at rotor inlet, tip radius
BETAHN	relative flow angle at rotor inlet, hub radius

WTN	relative velocity at rotor inlet, tip radius
WHN	relative velocity at rotor inlet, hub radius
BETATR	relative flow angle at rotor outlet, tip radius
BETAHR	relative flow angle at rotor outlet, hub radius
WTR	relative velocity at rotor outlet, tip radius
WHR	relative velocity at rotor outlet, hub radius
CHD	blade chord
ASRAT	aspect ratio
SPN	stator spacing
TBTAMN	tangent of BETAMN
TBTAMR	tangent of BETAMR
SPR	rotor spacing
TR	throat opening for the rotor
TN	throat opening for the stator
DR	hydraulic diameter for rotor
DN	hydraulic diameter for stator
RER	Reynolds number for rotor
REN	Reynolds number for stator
ZETAR2	Soderberg efficiency parameter corrected for aspect ratio for rotor
ZETAN2	Soderberg efficiency parameter corrected for aspect ratio for stator
ZETAR3	Soderberg efficiency parameter corrected for AR and Re, for rotor
ZETAN3	Soderberg efficiency parameter corrected for AR and Re, for stator
EFF	stage efficiency

ZX	number blades-decimal number
NZX	number blades-whole number
EFFX	machine efficiency
XMN	true Mach number at rotor inlet, hub radius, last stage
XMR	relative Mach number at rotor outlet, tip radius, last stage
REAH2	reaction at last stage, hub radius
ESTMX	centrifugal stress at blade root, last stage
DIAM1	inlet diameter
DIAM2	outlet diameter
UNLEN	machine length
G	radians to degrees conversion

Radial Turbine

Z	slip factor
RAD3	radius at rotor inlet
ALFA3	true inlet flow angle, rotor inlet
VM3	meridional velocity component at rotor inlet
V3	velocity at rotor inlet
XM3	Mach number at rotor inlet
FUN3	function f at rotor inlet (section 2.2.3)
AFLO3	flow area at rotor inlet
TR	stator blade height
RAD2	radius at stator exit
RRAT	radius ratio (section 2.4.4)

V4M	velocity at rotor outlet, mean radius
V4H	velocity at rotor outlet, hub radius
RAD4M	mean radius at rotor outlet
XM4M	Mach number at rotor outlet
FUN4	function f at rotor outlet (section 2.2.3)
AANN4	annulus area at rotor outlet
T4	blade height at rotor outlet
RAD4H	hub radius at rotor outlet
RAD4T	tip radius at rotor outlet
BB	exponential constant
DEN4	fluid static density at rotor outlet
Q	volumetric flow rate at rotor outlet
SPSP	non-dimensional specific speed
SPRR	stator outlet spacing
OP	stator throat opening
CR	stator chord
RAD1	stator inlet radius
SALFA1	sine of ALFA1
CALFA1	cosine of ALFA1
ASCR	maximum scroll area
V1	velocity at stator inlet
VM1	meridional velocity component at stator inlet
VT1	tangential
ALFA1	true flow angle at stator inlet
VT3	tangential velocity component at rotor inlet

U3	tangential rotor velocity at inlet
BETA3	relative flow angle at rotor inlet
W3	relative velocity at rotor inlet
U4M	tangential rotor velocity at outlet, mean radius
U4H	tangential rotor velocity at outlet, hub radius
U4T	tangential rotor velocity at outlet, tip radius
BETA4M	relative flow angle at rotor outlet, mean radius
BETA4H	relative flow angle at rotor outlet, hub radius
BETA4T	relative flow angle at rotor outlet, tip radius
W4M	relative velocity at rotor outlet, mean radius
W4H	relative velocity at rotor outlet, hub radius
W4T	relative velocity at rotor outlet, tip radius
XM4	relative Mach number at rotor outlet, tip radius
RE3	Reynolds number at rotor inlet
DEN3	fluid static density at rotor inlet
XLM	windage loss
XLC	clearance loss
SPRM	mean rotor spacing
SIGMAR	solidity
ER	rotor boundary layer loss coefficient
XLR	rotor boundary layer loss
ALFAO	stator angle (section 2.4.7)
ALFAST	stator stagger angle
ES	stator boundary layer loss coefficient
XLS	stator boundary layer loss

EFFR	machine efficiency
UNLENR	machine length
DIA3	diameter at rotor inlet
DIA4T	tip diameter at rotor outlet
DIA4M	hub diameter at rotor outlet
DIA1	diameter at stator inlet
DIA4M	mean diameter at rotor outlet
DIA2	diameter at stator outlet

SUBPROGRAM SODCOR

X1	gas deflection angle, radians
X	gas deflection angle, degrees
XX	abscissa for Soderberg correlation
YY	ordinate
XL1	first LaGrange Polynomial
XL2	second LaGrange Polynomial
XL3	third LaGrange Polynomial
Y	Soderberg gas deflection loss parameter

SUBPROGRAM FLOREL

GAMA	ratio of specific heats
R	universal gas constant
CP	specific heat at constant pressure
TO	stagnation temperature
V	velocity

XM	Mach number
FLFN	function f (section 2.2.3)
T	static temperature
EXPR	internal constant
CON	internal constant
FUN	internal constant

APPENDIX B

B.1 Main Program

A complete listing of the main program is contained on the following pages. Appendix E contains detailed instructions for input-output and running the program. The mathematical model for the program is developed in chapter 2. The logic sequence used in programming the model is outlined below:

1) Definition of real variables and common variables
2) Dimensioning of subscripted variables
3) Input format specification
4) Output format specification
5) Readin input data
6) Readout input data for checking
7) Convert input data to standard units
8) Calculate fluid properties
9) Readout fluid properties for checking
10) Commence do-loop for calculations on each set of turbine input parameters
11) Calculate axial turbine mean parameters
12) Calculate axial first and last stage parameters on internal do-loop
13) If design is rejected on specified checks, axial calculations are dropped and radial are commenced.
14) Calculate axial machine parameters
15) Convert axial variables to output units
16) Readout output data for axial machine
17) Calculate radial parameters
18) If design is rejected on specified checks, radial calculations are dropped and next set of axial calculations commenced
19) Convert radial variables to output units
20) Readout output data for radial machine
21) Commence calculations for next set of axial input parameters

```
C     TURBINE DESIGN PROGRAM
C     SUBPROGRAMS SODCOR AND FLOREL MUST BE LOADED ALSO
      REAL NU(2)
      DIMENSION BETAHR(2),WTR(2),WHR(2),BETATR(2),WTHN(2),BETATN(2),BETA
     1HN(2),WTN(2),WHN(2),Y(5),WTTN(2),TO(2),PO(2),NB(2),ZX(2),FLFN(2),A
     2FLO(2),AANN(2),T(2),RADH(2),EFF(2),RADT(2),ASRAT(2),VTTN(2),A(5),B
     3(5),C(5),D(5),E(5),F(5),VTHN(2),CSTX(5),ALFATN(2),ALFAHN(2),VTN(2)
     4,VHN(2),UTN(2),UHN(2),NZX(2)
      COMMON GAMA,R,CP
 1000 FORMAT(4(F10.5,10X))
 1010 FORMAT(10X,F5.0,5X,F5.2,5X,F10.0,10X,F5.2,5X,F5.0)
 1020 FORMAT(10X,F10.0,10X,F5.0,5X,F5.0,5X,F10.2)
 1040 FORMAT(I2)
 1050 FORMAT(5X,'                                                       ')
 2000 FORMAT(5X,'CP',2X,F6.4,5X,'GAMMA',2X,F7.4,5X,'R',2X,F7.3)
 2010 FORMAT(5X,'UNFEASIBLE AXIAL DESIGN, GAS DEFLECTION TOO BIG')
 2020 FORMAT(5X,'UNFEASIBLE AXIAL DESIGN, HUB RADIUS TOO SMALL')
 2030 FORMAT(5X,'UNFEASIBLE AXIAL DESIGN, HUB REAL MACH NO. TOO BIG')
 2040 FORMAT(5X,'UNFEASIBLE AXIAL DESIGN, TIP REL MACH NO. TOO BIG')
 2050 FORMAT(5X,'UNFEASIBLE AXIAL DESIGN, HUB REACTION TOO SMALL')
 2060 FORMAT(5X,'UNFEASIBLE RADIAL DESIGN, SPECIFIC SPEED TOO RADICAL')
 2070 FORMAT(5X,'LAST STAGE TIP REL MACH NO.',2X,F5.3,5X,'LAST STAGE HUB
     1 REAL MACH NO.',2X,F5.3)
 2090 FORMAT(5X,'FIRST STAGE TIP DIAM.',2X,F5.2,5X,'BLADE HEIGHT',F7.4,5
     1X,'NO BLADES',2X,F5.0,5X,'EFF.',2X,F7.4)
 2110 FORMAT(5X,'LAST STAGE TIP DIAM.',2X,F5.2,5X,'BLADE HEIGHT',2X,F7.4
     1,5X,'NO. BLADES',2X,F5.0,5X,'EFF.',2X,F7.4)
 2130 FORMAT(5X,'MACHINE EFFICIENCY',2X,F7.4)
 2140 FORMAT(5X,'MAX CENT. STRESS',5X,F10.2,5X,'MAX GAS BENDING STRESS 1
     1 TPI')
 2150 FORMAT(5X,'UNIT LENGTH',F10.3)
 2160 FORMAT(1H+/////5X,'AXIAL TURBINE'/)
 2170 FORMAT(1H+/////5X,'RADIAL INFLOW TURBINE'/)
 2180 FORMAT(5X,'NOZZLE INLET DIAM.',2X,F7.2,5X,'BLADE HEIGHT',2X,F9.4)
 2190 FORMAT(5X,'ROTOR INLET DIAM.',2X,F7.2,5X,'OUTLET TIP DIAM.',2X,F7.
     12,5X,'HUB DIAM.',2X,F7.2)
```

```
2200 FORMAT(5X,'MAX SCROLL AREA',2X,F10.3)
2210 FORMAT(5X,'SPECIFIC SPEED',F10.6)
2220 FORMAT(5X,'EFFICIENCY',2X,F7.4/)
2230 FORMAT(5X,'UNIT LENGTH',F7.2)
2240 FORMAT(5X,'ASPECT RATIO FIRST STAGE',2X,F5.2,5X,'LAST STAGE',2X,F5
    1.2/)
2250 FORMAT(5X,'VELOCITY TRIANGLES')
2260 FORMAT(5X,'HUB')
2270 FORMAT(5X,'UHN',2X,F7.1,5X,'VTHN',2X,F7.1,5X,'VX',2X,F7.1,5X,'VHN'
    1,2X,F7.1,5X,'WHN',2X,F7.1,5X,'WHR',2X,F7.1)
2280 FORMAT(5X,'ALFAHN',2X,F5.1,5X,'BETAHN',2X,F5.1,5X,'BETAHR',2X,F5.1
    1)
2290 FORMAT(5X,'MEAN')
2300 FORMAT(5X,'UMN',2X,F7.1,5X,'VTMN',2X,F7.1,5X,'VX',2X,F7.1,5X,'VMN'
    1,2X,F7.1,5X,'WMN',2X,F7.1,5X,'WMR',2X,F7.1)
2310 FORMAT(5X,'ALFAMN',2X,F5.1,5X,'BETAMN',2X,F5.1,5X,'BETAMR',2X,F5.1
    1)
2320 FORMAT(5X,'TIP')
2330 FORMAT(5X,'UTN',2X,F7.1,5X,'VTTN',2X,F7.1,5X,'VX',2X,F7.1,5X,'VTN'
    1,2X,F7.1,5X,'WTN',2X,F7.1,5X,'WTR',2X,F7.1)
2340 FORMAT(5X,'ALFATN',2X,F5.1,5X,'BETATN',2X,F5.1,5X,'BETATR',2X,F5.1
    1)
2350 FORMAT(1H1)
2360 FORMAT(5X,'FIRST STAGE')
2370 FORMAT(5X,'LAST STAGE')
2380 FORMAT(5X,'NOZZLE OUTLET DIAM.',2X,F7.2)
2390 FORMAT(5X,'NOZZLE INLET')
2400 FORMAT(5X,'V1',2X,F7.1,5X,'VT1',2X,F7.1,5X,'VM1',2X,F7.1,5X,'ALFA1
    1',2X,F5.1)
2410 FORMAT(5X,'ROTOR INLET')
2420 FORMAT(5X,'U3',2X,F7.1,5X,'VT3',2X,F7.1,5X,'VM3',2X,F7.1,5X,'V3',2
    1X,F7.1,5X,'W3',2X,F7.1)
2430 FORMAT(5X,'ALFA3',2X,F5.1,5X,'BETA3',2X,F5.1)
2440 FORMAT(5X,'ROTOR OUTLET')
2450 FORMAT(5X,'U4H',2X,F7.1,5X,'V4H',2X,F7.1,5X,'W4H',2X,F7.1,5X,'BETA
    14H',2X,F5.1)
```

```
2460 FORMAT(5X,'U4M',2X,F7.1,5X,'V4M',2X,F7.1,5X,'W4M',2X,F7.1,5X,'BETA
    14M',2X,F5.1)
2470 FORMAT(5X,'U4T',2X,F7.1,5X,'V4T',2X,F7.1,5X,'W4T',2X,F7.1,5X,'BETA
    14T',2X,F5.1)
2490 FORMAT(5X,'ASSUMED',5X,F5.0,2X,'STAGES',5X,F5.2,2X,'REACTION AT ME
    1AN',5X,F10.0,2X,'RPM')
2500 FORMAT(5X,'ASSUMED',5X,F5.2,2X,'ALFAMN',5X,F5.0,2X,'UNITS',5X,F10.
    12,2X,'WW')
2510 FORMAT(5X,'ASSUMED',5X,F10.0,2X,'RPM',5X,F5.0,2X,'UNITS',5X,F5.0,2
    1X,'BLADES',5X,F10.2,2X,'WW')
2520 FORMAT(5X,'UNFEASIBLE RADIAL DESIGN, INLET MACH NO. TOO HIGH')
2530 FORMAT(5X,'UNFEASIBLE RADIAL DESIGN, OUTLET RELATIVE MACH NO. TOO
    1HIGH')
C     INPUT DATA
      READ(8,1000)HO1,TO(1),PO(1),NU(1)
      READ(8,1000)HO2,TO(2),PO(2),NU(2)
      READ(8,1000)HO2S,TO2S,PO2S
      WRITE(5,1000)HO1,TO(1),PO(1),NU(1)
      WRITE(5,1000)HO2,TO(2),PO(2),NU(2)
      WRITE(5,1000)HO2S,TO2S,PO2S
      DO 10 I=1,2
      PO(I)=PO(I)*144.
      NU(I)=NU(I)/3600.
   10 CONTINUE
      PO2S=PO2S*144.
C     GAS PROPERTIES
      CP=(HO1-HO2S)/(TO(1)-TO2S)
      GAMA=ALOG(PO2S/PO(1))/(ALOG(PO2S/PO(1))-ALOG(TO2S/TO(1)))
      CV=CP/GAMA
      R=CP-CV
      DHO=HO1-HO2
      RJ=R*778.
      WRITE(5,2000)CP,GAMA,RJ
      WRITE(5,2350)
      READ(8,1040)MM
      DO 999 NN=1,MM
```

```
      READ(8,1050)
      WRITE(5,1050)
      READ(8,1010)AXN,REAM,RPMX,ALFAMN,UNITX
      READ(8,1020)RPMR,UNITR,ZR,WW
      WRITE(5,2490)AXN,REAM,RPMX
      WRITE(5,2500)ALFAMN,UNITX,WW
      ALFAMN=ALFAMN/57.295778
C     AXIAL TURBINE
C     MEAN PARAMETERS
      IF(ABS(1.-REAM).LE..01) REAM=.99
      RADM=(1068.247/RPMX)*SQRT(DHO/(AXN*(1.-REAM)))
      VTMN=223.7331*SQRT(DHO*(1.-REAM)/AXN)
      VMN=VTMN/SIN(ALFAMN)
      VX=VTMN*COS(ALFAMN)/SIN(ALFAMN)
      DUM=(VX*VX)+((RPMX*RADM*.1047197)**2.)
      WMR=SQRT(DUM)
      TALFMN=SIN(ALFAMN)/COS(ALFAMN)
      DUM=2.*(1.-REAM)
      BETAMN=ATAN((TALFMN)*(1.-(1./DUM)))
      BETAMR=ATAN((TALFMN)/DUM)
      CALL SODCOR(ALFAMN,ZETAN1)
      DUM=BETAMN+BETAMR
      CALL SODCOR(DUM,ZETAR1)
      CON=RADM*VTMN
      IF(ZETAN1.EQ.5.) GO TO 15
      IF(ZETAR1.EQ.5.) GO TO 15
      GO TO 20
   15 WRITE(5,2010)
      GO TO 500
   20 CONTINUE
      WMN=VX/COS(BETAMN)
      UMN=.1047197*RPMX*RADM
C     FIRST AND LAST STAGE PARAMETERS
      DO 100 J=1,2
      CALL FLOREL(TO(J),VMN,DUM,FLFN(J))
      AFLO(J)=FLFN(J)*SQRT(TO(J))*WW/(UNITX*PO(J))
```

```
      AANN(J)=AFLO(J)/COS(ALFAMN)
      T(J)=AANN(J)/(6.2831854*RADM)
      RADT(J)=RADM+(T(J)/2.)
      RADH(J)=RADM-(T(J)/2.)
      IF(RADH(J).LT.(.5*RADT(J))) GO TO 50
      GO TO 60
   50 WRITE(5,2020)
      GO TO 500
   60 CONTINUE
      VTTN(J)=CON/RADT(J)
      VTHN(J)=CON/RADH(J)
      ALFATN(J)=ATAN(VTTN(J)/VX)
      ALFAHN(J)=ATAN(VTHN(J)/VX)
      VTN(J)=VX/COS(ALFATN(J))
      VHN(J)=VX/COS(ALFAHN(J))
      UTN(J)=.1047197*RPMX*RADT(J)
      UHN(J)=.1047197*RPMX*RADH(J)
      WTTN(J)=VTTN(J)-UTN(J)
      WTHN(J)=VTHN(J)-UHN(J)
      BETATN(J)=ATAN(WTTN(J)/VX)
      BETAHN(J)=ATAN(WTHN(J)/VX)
      WTN(J)=VX/COS(BETATN(J))
      WHN(J)=VX/COS(BETAHN(J))
      BETATR(J)=ATAN(UTN(J)/VX)
      BETAHR(J)=ATAN(UHN(J)/VX)
      WTR(J)=VX/COS(BETATR(J))
      WHR(J)=VX/COS(BETAHR(J))
      DUM=(SIN(BETAHN(J))/COS(BETAHN(NJ)))+(SIN(BETAHR(J))/COS(BETAHR(J)
     1))
      DUM=(DUM*5.0996*(COS(BETAHR(J))**2.))+(2.3796*SIN(BETAMN)/COS(BETA
     1MN))
      DUM=DUM/(32.174*RPMX*RADM*RADH(J)*AXN)
      DUM=DUM*T(J)*DHO*WW
      CHD=SQRT(DUM)
      ASRAT(J)=T(J)/CHD
      IF(ASRAT(J).GT.10.) GO TO 65
```

```
      IF(ASRAT(J).LT.0.25) GO TO 70
      GO TO 75
   65 ASRAT(J)=10.
      GO TO 75
   70 ASRAT(J)=.25
   75 CONTINUE
      CHD=T(J)/ASRAT(J)
      SPN=.4*CHD/(COS(ALFAMN)*SIN(ALFAMN))
      TBTAMN=SIN(BETAMN)/COS(BETAMN)
      TBTAMR=SIN(BETAMR)/COS(BETAMR)
      SPR=.4*CHD/(COS(BETAMR)*COS(BETAMR)*(TBTAMN+TBTAMR))
      TR=SPR*COS(BETAMR)
      TN=SPN*COS(BETAMN)
      DR=2.*TR*T(J)/(TR+T(J))
      DN=2.*TN*T(J)/(T(J)+TN)
      RER=DR*WMR/NU(J)
      REN=DN*VMN/NU(J)
      ZETAR2=((1.+ZETAR1)*(.975+(.075*CHD/T(J))))-1.
      ZETAN2=((1.+ZETAN1)*(.975+(.075*CHD/T(J))))-1.
      ZETAR3=ZETAR2*((100000./RER)**.25)
      ZETAN3=ZETAN2*((100000./REN)**.25)
      DUM1=ZETAN3*VMN*VMN/(2.*UMN*VTMN)
      DUM2=ZETAR3*WMR*WMR/(2.*UMN*VTMN)
      EFF(J)=.99/(1.+DUM1+DUM2)
      ZX(J)=6.2831854*RADH(J)/SPN
      NZX(J)=IFIX(ZX(J))
      ZX(J)=FLOAT(NZX(J))
  100 CONTINUE
C     EFFICIENCY,MACH NUMBERS AND HUB REACTION CHECK
      EFFX=(EFF(1)+EFF(2))/2.
      CALL FLOREL(TO(2),VHN(2),XMN,DUM)
      CALL FLOREL(TO(2),WTR(2),XMR,DUM)
      IF(XMN.GT.0.98) GO TO 110
      IF(XMR.GT.0.98) GO TO 120
      GO TO 130
  110 WRITE(5,2030)
```

```
      GO TO 500
  120 WRITE(5,2040)
      GO TO 500
  130 CONTINUE
      REAH2=1.-(VTHN(2)/(2.*UHN(2)))
      IF(REAH2.LT.0.01) GO TO 140
      GO TO 150
  140 WRITE(5,2050)
      GO TO 500
  150 CONTINUE
C     STRESSES AND SIZES
      CSTMX=0.001186*(RPMX**2.)*RADM*T(2)
      DIAM1=2.*RADT(1)
      DIAM2=2.*RADT(2)
      UNLEN=AXN*((T(1)/ASRAT(1))+CHD)/2.
C     OUTPUT
      WRITE(5,2160)
      WRITE(5,2070)XMR,XMN
      WRITE(5,2090)DIAM1,T(1),ZX(1),EFF(1)
      WRITE(5,2110)DIAM2,T(2),ZX(2),EFF(2)
      WRITE(5,2130)EFFX
      WRITE(5,2140)CSTMX
      WRITE(5,2150)UNLEN
      WRITE(5,2240)ASRAT(1),ASRAT(2)
      WRITE(5,2250)
      G=57.295778
      ALFAMN=ALFAMN*G
      BETAMN=BETAMN*G
      BETAMR=BETAMR*(-G)
      DO 300 J=1,2
      ALFAHN(J)=ALFAHN(J)*G
      BETAHN(J)=BETAHN(J)*G
      BETAHR(J)=BETAHR(J)*(-G)
      ALFATN(J)=ALFATN(J)*G
      BETATN(J)=BETATN(J)*G
      BETATR(J)=BETATR(J)*(-G)
```

```
      IF(J.EQ.1) WRITE(5,2360)
      IF(J.EQ.2) WRITE(5,2370)
      WRITE(5,2260)
      WRITE(5,2270)UHN(J),VTHN(J),VX,VHN(J),WHN(J),WHR(J)
      WRITE(5,2280)ALFAHN(J),BETAHN(J),BETAHR(J)
      WRITE(5,2290)
      WRITE(5,2300)UMN,VTMN,VX,VMN,WMN,WMR
      WRITE(5,2310)ALFAMN,BETAMN,BETAMR
      WRITE(5,2320)
      WRITE(5,2330)UTN(J),VTTN(J),VX,VTN(J),WTN(J),WTR(J)
      WRITE(5,2340)ALFATN(J),BETATN(J),BETATR(J)
  300 CONTINUE
  500 WRITE(5,2350)
      WRITE(5,1050)
      WRITE(5,2510)RPMR,UNITR,ZR,WW
C     RADIAL TURBINE
C     ROTOR
      G=57.295778
      Z=1.-(2./ZR)
      RAD3=1510.8233*SQRT(DHO/(Z*RPMR*RPMR))
      ALFA3=ATAN((ZR-2.)/6.2831854)
      VM3=994.08186*SQRT(DHO/Z)/ZR
      V3=VM3/COS(ALFA3)
      CALL FLOREL(TO(1),V3,XM3,FUN3)
      AFLO3=WW*SQRT(TO(1))*FUN3/(UNITR*PO(1))
      TR=AFLO3/(6.2831854*RAD3*COS(ALFA3))
      RAD2=RAD3+2.*TR
      RPAT=6.0
  600 V4M=9.134702*SQRT(DHO/Z)*RPAT
      V4H=V4M
      RAD4M=RAD3*RPAT/10.0
      CALL FLOREL(TO(2),V4H,XM4M,FUN4)
      AANN4=WW*SQRT(TO(2))*FUN4/(UNITR*PO(2))
      T4=AANN4/(6.2831854*RAD4M)
      RAD4H=RAD4M-(T4/2.)
      RAD4T=RAD4M+(T4/2.)
```

```
      IF(RAD4H.LT.(0.1*RAD3))GO TO 650
      GO TO 700
  650 RRAT=RRAT+0.2
      GO TO 600
C     CHECK SPECIFIC SPEED
  700 BB=1./(GAMA-1.)
      DEN4=(PO(2)/(RJ*TO(2)))*((1.-((GAMA-1.)*(XM4M**2.)/2.))**BB)
      Q=WW/(DEN4*UNITR)
      SPSP=RPMR*SQRT(Q)/(60.*(25031.372*DHO)**.75)
      IF(SPSP.LT.0.05) GO TO 550
      IF(SPSP.GT.0.25) GO TO 550
      GO TO 560
  550 WRITE(5,2060)
      GO TO 998
  560 CONTINUE
C     NOZZLE PARAMETERS
      SPRR=6.2831854*RAD2/ZR
      OP=SPRR*COS(ALFA3)
      CR=SPRR/.6
      DUM1=RAD2+(CR*COS((3.1415927/ZR)+ALFA3))
      DUM2=CR*SIN((3.1415927/ZR)+ALFA3)
      RAD1=SQRT((DUM1**2.)+(DUM2**2.))
C     MAX SCROLL AREA
      SALFA1=(RAD2/RAD1)*SIN((3.1415927*(1.-(1./ZR)))-ALFA3)
      CALFA1=SQRT(1.-(SALFA1*SALFA1))
      ASCR=6.2831854*RAD1*TR*CALFA1
C     VELOCITY TRIANGLES
      V1=V3*RAD3/RAD1
      VM1=V1*CALFA1
      VT1=V1*SALFA1
      ALFA1=ATAN(SALFA1/CALFA1)
      VT3=V3*SIN(ALFA3)
      U3=0.1047197*RPMR*RAD3
      BETA3=ATAN((VT3-U3)/VM3)
      W3=VM3/COS(BETA3)
      U4M=0.1047197*RPMR*RAD4M
```

```
      U4H=0.1047197*RPMR*RAD4H
      U4T=0.1047197*RPMR*RAD4T
      BETA4M=ATAN(-U4M/V4M)
      BETA4H=ATAN(-U4H/V4M)
      BETA4T=ATAN(-U4T/V4M)
      W4M=V4M/COS(BETA4M)
      W4H=V4M/COS(BETA4H)
      W4T=V4M/COS(BETA4T)
      CALL FLORFL(TO(2),W4T,XM4,DUM)
      IF(XM3.GT.1.1) GO TO 710
      IF(XM4.GT.1.1) GO TO 720
      GO TO 750
  710 WRITE(5,2520)
      GO TO 998
  720 WRITE(5,2530)
      GO TO 998
  750 CONTINUE
C     EFFICIENCY
      RE3=.2094395*RPMR*(RAD3**2.)/NU(1)
      DEN3=(PO(1)/(RJ*TO(1)))*((1.-((GAMA-1.)*(XM3**2.)/2.))**BB)
      XLW=(DEN3**2.)*NU(2)*UNITR*(RAD3**2.)*(2.24)/((RE3**.2)*WW*(10.**6
     1.))
      XLC=(DHO/2.)*((.004*RAD3/TR)+(.005*RAD4T/(RAD4T-RAD4H)))
      SPRM=(3.1415927/ZR)*(RAD3+RAD4H+RAD4T)
      DUM=((RAD4M**2.)-(RAD3*RAD4M))/((RAD3**2.)+(2.*RAD4M*RAD3))
      SIGMAR=ABS(0.5092958*ZR*DUM)
      ER=.017*SIGMAR*(1.+(1.9*SPRM/(TR+RAD4T-RAD4H)))/(.5-(.003*ZR)-(.01
     17*SIGMAR))
      XLR=ER*(W4M**2.)/(50062.744*(1.-ER))
      ALFAO=ATAN(SIN(ALFA3)/((2.*TR/RAD3)+COS(ALFA3)))
      ALFAST=(ALFAO+ALFA3)/2.
      ES=.0076*(1.+(COS(ALFAST)/.7))/(COS(ALFA3)-.025)
      XLS=ES*(V3**2.)/(50062.744*(1.-ES))
      EFFR=(DHO-XLW-XLC)/(DHO+XLS+XLR)
      UNLENR=RAD3-RAD4H
      DIA3=2.*RAD3
```

```
      DIA4T=2.*RAD4T
      DIA4H=2.*RAD4H
      DIA1=2.*RAD1
      DIA4M=2.*RAD4M
      DIA2=2.*RAD2
      ALFA1=ALFA1*G
      ALFA3=ALFA3*G
      BETA3=BETA3*G
      BETA4H=BETA4H*G
      BETA4M=BETA4M*G
      BETA4T=BETA4T*G
C     OUTPUT
      WRITE(5,2170)
      WRITE(5,2180)DIA1,TR
      WRITE(5,2380)DIA2
      WRITE(5,2190)DIA3,DIA4T,DIA4H
      WRITE(5,2230)UNLENR
      WRITE(5,2200)ASCR
      WRITE(5,2210)SPSP
      WRITE(5,2220)EFFR
      WRITE(5,2250)
      WRITE(5,2390)
      WRITE(5,2400)V1,VT1,VM1,ALFA1
      WRITE(5,2410)
      WRITE(5,2420)U3,VT3,VM3,V3,W3
      WRITE(5,2430)ALFA3,BETA3
      WRITE(5,2440)
      WRITE(5,2260)
      WRITE(5,2450)U4H,V4H,W4H,BETA4H
      WRITE(5,2290)
      WRITE(5,2460)U4M,V4H,W4M,BETA4M
      WRITE(5,2320)
      WRITE(5,2470)U4T,V4H,W4T,BETA4T
  998 WRITE(5,2350)
  999 CONTINUE
      END
```

APPENDIX C

C Subprogram SODCOR

This program computes an efficiency parameter from given data points using LaGrange Polynomials in a curve-fitting logic. A complete listing of the subprogram is contained on the following page. The mathematical model is explained in section 3.4. The logic sequence used in programming the model is outlined below:

1) Transfer of input variables
2) Definition of curve data points
3) Conversion of input data to standard units
4) Classification of input data as to sub-division of curve for curve-fitting purposes
5) Definition of sub-division curve-fitting data
6) Calculation of LaGrange Polynomials
7) Calculation of output variable
8) Transfer of output variable

```
      SUBROUTINE SODCOR (X1,Y)
      DIMENSION XX(15),YY(15)
      DATA XX(1)/.01/,XX(2)/10./,XX(3)/20./,XX(4)/30./,XX(5)/40./,XX(6)/
     150./,XX(7)/60./,XX(8)/70./,XX(9)/80./,XX(10)/90./,XX(11)/100./,YY(
     21)/.0424/,YY(2)/.0452/,YY(3)/.0476/,YY(4)/.0508/,YY(5)/.0548/,YY(6
     3)/.0592/,YY(7)/.0644/,YY(8)/.0696/,YY(9)/.0788/,YY(10)/.0892/,YY(1
     41)/.1/,XX(12)/110./,XX(13)/120./,XX(14)/130./,XX(15)/140./
      DATA YY(12)/.1144/,YY(13)/.135/,YY(14)/.1564/,YY(15)/.1794/
      X=X1*57.295778
      IF(X.LE.20.) GO TO 10
      IF(X.LE.40.) GO TO 20
      IF(X.LE.60.) GO TO 30
      IF(X.LE.80.) GO TO 40
      IF(X.LE.100.) GO TO 50
      IF(X.LE.120.)GO TO 60
      IF(X.LE.140.)GO TO 70
      IF(X.GT.140.)GO TO 80
   10 N=1
      GO TO 100
   20 N=3
      GO TO 100
   30 N=5
      GO TO 100
   40 N=7
      GO TO 100
   50 N=9
      GO TO 100
   60 N=11
      GO TO 100
   70 N=13
  100 XL1=((X-XX(N+1))*(X-XX(N+2)))/((XX(N)-XX(N+1))*(XX(N)-XX(N+2)))
      XL2=((X-XX(N))*(X-XX(N+2)))/((XX(N+1)-XX(N))*(XX(N+1)-XX(N+2)))
      XL3=((X-XX(N))*(X-XX(N+1)))/((XX(N+2)-XX(N))*(XX(N+2)-XX(N+1)))
      Y=(XL1*YY(N))+(XL2*YY(N+1))+(XL3*YY(N+2))
      RETURN
   80 Y=5.
      RETURN
      END
```

APPENDIX D

D Subprogram FLOREL

This subprogram computes a flow function and Mach number using the one-dimensional flow functions. The mathematical model is explained in section 2.2.3. A complete listing of the program is contained on the following page. The logic sequence used in programming the model is outlined below.

1) Transfer of input variables
2) Transfer of common variables
3) Calculation of Mach number
4) Calculation of flow function
5) Transfer of output variables

```
SUBROUTINE FLOREL(TO,V,XM,FLFN)
COMMON GAMA,R,CP
T=TO-(V*V/(50062.744*CP))
DUM=GAMA*R*T*25031.372
XM=V/SQRT(DUM)
EXPR=(GAMA+1.)/(2.*GAMA-2.)
CON=SQRT(GAMA*2.0413547/R)
FUN=1.+(((GAMA-1.)/2.)*XM*XM)
FLFN=(FUN**EXPR)/(XM*CON)
RETURN
END
```

APPENDIX E

E.1 Specific Instructions for Use

This section is intended to provide detailed instructions for the use of the computer programs described in the previous appendices including input-output interpretation. The specific program control cards described are required on the Interdata 70 computer. Check the user's manual before running on other machines as these control cards may differ.

E.2 Loading Sequence

The sequence for loading the main and subprograms with their required control cards is as follows:

1) Job card
2) Language card
3) Listing designation card
4) Main program
5) Subprogram SODCOR
6) Subprogram FLOREL
7) Execution card
8) Common length designation card
9) Data cards
10) Termination card

E.3 Job Control Cards

The JOB card, language card, listing designation card and JOB termination cards are all of the standard format, common to any job run on the Interdata 70 system. The execution card must carry a numeral one in column seventeen in addition to the standard format. The common length designation card specifies the space in blank common to be used in the job. The format for this card is *BC 000C commencing

in column one.

E.4 Data Cards

E.4.1 General

The Data cards necessary for use of the program can be grouped into three categories.

1) Cycle Data
2) Number of Design Sets
3) Turbine Design input sets

They must be loaded in the above order. Card groups 1 and 2 are only required once per job, while card group 3 is required for each individual design set examined.

E.4.2 Cycle Data

Three cards are required to specify the data for a given thermal cycle. The context of each is given below. No information can be omitted, and they must be loaded in the order described.

SPACES	INFORMATION	UNITS	FORMAT
CARD ONE			
1-10	inlet total enthalpy	BTU/lbm	F10.5
20-30	inlet total temperature	^oR	F10.5
40-50	inlet total pressure	psia	F10.5
60-70	inlet kinematic viscosity	ft^2/hr	F10.5
CARD TWO			
1-10	outlet total enthalpy	BTU/lbm	F10.5
20-30	outlet total temperature	^oR	F10.5
40-50	outlet total pressure	psia	F10.5
60-70	outlet kinematic viscosity	ft^2/hr	F10.5

SPACES	INFORMATION	UNITS	FORMAT
CARD THREE			
1-10	outlet isentropic total enthalpy	BTU/lbm	F10.5
20-30	outlet isentropic total temperature	^{o}R	F10.5
40-50	outlet isentropic total pressure	psia	F10.5

E.4.3 Number of Design Sets

One card is necessary to specify the number of sets of turbine parameters entered. A set consists of one combination of axial turbine input parameters, and one combination of radial turbine input parameters, with both combinations sharing the same total mass flow rate (section 2.2.2). The program will accommodate up to 99 separate sets of input data. The number of sets must be specified in I2 format on spaces one and two of the card with the other spaces left blank. This card may not be omitted.

E.4.4 Turbine Design Input Sets

A turbine design input set consists of a combination of axial input parameters, section 2.3.3, and a combination of radial input parameters, section 2.4.3, which share the same total mass flow rate, section 2.2.2. Three cards are necessary for each set and no information or card may be omitted. They must be loaded in the order in which they are described. In the event that a run is not to be designated by name, a blank card must be included in place of card one.

SPACES	INFORMATION	UNITS	FORMAT
CARD ONE			
6-70	Any desired run designation information		H
CARD TWO-Axial Turbine			
11-15	Number of stages		F5.0
21-25	Inlet reaction at midblade		F5.2
31-40	Shaft RPM	Rev/min	F10.0
51-55	Rotor inlet flow angle at midblade	Degrees	F5.2
61-65	Number of machines operating in parallel on total flow path (may be one)		F5.0
CARD THREE-Radial Inflow Turbine			
11-20	Shaft RPM	Rev/min	F10.0
31-35	Number of machines operating in parallel on total flow path (may be one)		F5.0
41-45	Number of rotor blades		F5.0
51-60	Total mass flow rate for all machines-i.e. sum of all parallel flow paths. See section 2.2.2. It must be the same for Axial and Radial Design Set.	lbm/sec	F10.2

E.5 Output

E.5.1 General

The cycle data will be printed out once for each job, in the same format it was entered in with, as a check for accuracy. The universal gas constant, in units of ft lbf/ lbm^{o}R, the specific heat at constant pressure, and the ratio of specific heats will also be printed out as a check. For

each turbine design data set, the run designation information, the assumed input variables, and the preliminary design data will be printed out. If the assumed input variables result in a design which doesn't pass the axial or radial feasibility checks, sections 2.3.8 and 2.4.9, the design data will be omitted and an unfeasibility message printed, section E.5.3.

E.5.2 Preliminary Design Data

The format of the preliminary design data is self-explanatory. The output is in units of feet for all dimensions, degrees for all angles, and feet per second for all velocities. Stress is given in psi and tpi. The abbreviations used in describing the velocity triangles are as follows:

Axial Turbine

UHN, UMN, UTN, - rotor tangential velocity at hub, mean and tip radii

VTHN,VTMN,VTTN,- fluid tangential velocity component at rotor inlet at hub, mean and tip radii

VX - fluid axial velocity component

VHN, VMN, VTN, - fluid velocity at rotor inlet at hub, mean and tip radii

WHN, WMN, WTN, - fluid relative velocity at rotor inlet at hub, mean and tip radii

WHR, MMR, WTR, - fluid relative velocity at rotor outlet, hub, mean and tip radii

ALFAHN,ALFAMN,ALFATN - rotor inlet flow angle at hub, mean and tip radii

BETAHN,BETAMN,BETATN, - relative rotor inlet flow angle at hub, mean and tip radii

BETAHR,BETAMR,BETATR, - relative rotor outlet flow angle at hub, mean and tip radii

Radial Turbine

V1, V3 - fluid velocity at station 1 or 3

VT1, VT3, - fluid tangential velocity component at station 1 or 3

VM1, VM3, - fluid meridional velocity component at station 1 or 3

U3 - rotor tangential velocity at inlet

W3 - fluid relative velocity at rotor inlet

ALFA1,ALFA3, - inlet flow angle at station 1 or 3

BETA3 - relative rotor inlet flow angle

U4H,U4M,U4T, - rotor outlet tangential velocity at hub, mean and tip radii

V4H,V4M,V4T, - rotor outlet fluid velocity at hub, mean and tip radii

W4H,W4M,W4T, - rotor outlet fluid relative velocity at hub, mean and tip radii

BETA4H,BETA4M,BETA4T, - rotor outlet fluid relative flow angle at hub, mean and tip radii

E.5.3 Unfeasibility Conditions

If a given set of turbine design inputs does not meet the feasibility checks described in sections 2.3.8 and 2.4.9, a design unfeasibile message will be printed. As a reference, the checks associated with each message are summarized below.

MESSAGE	INTERPRETATION
Axial Turbine	
Gas Deflection too big	Either α_1 or $(\beta_1 + \beta_2)$ is greater than 140^0

MESSAGE	INTERPRETATION
Hub Radius too small	$r_h < 0.5\ r_t$
Hub Real Mach Number too big	Last stage hub true Mach number greater than 0.99
Tip Real Mach Number too big	Last stage tip relative Mach number greater than 0.99
Hub reaction too small	Last stage hub reaction less than 0.01
<u>Radial Turbine</u>	
Specific Speed to Radical	Machine specific speed less than 0.05 or greater than 0.25.
Inlet Mach number too high	Mach number at rotor inlet greater than 1.1.
Outlet Relative Mach number too high	Relative Mach number at rotor tip greater than 1.1

E.5.4 Compilation Errors

It is possible, given certain sets of axial turbine input data, to cause a computer run time error (negative square root and overflow). This condition sometimes occurs during the calculation of aspect ratio. It has been forseen in programming the model and will be internally corrected if it occurs. The data output is still valid if this error occurs even though an error message is printed out. Other run time errors or errors in computing the radial turbine data should be cause for re-examination of the input data provided.

APPENDIX F

EVALUATION OF THE ZENER CYCLE

F.1 General

The Zener Cycle, as described by Clarence Zener (15), is based upon the thermocline existing in the oceans of the world. According to Zener, it should be possible to take advantage of this temperature difference, (20°C is used as an example) to power a thermal cycle which could in turn produce useful energy. This supposition is based upon the assumption that technology could produce the necessary equipment. Assuming that this is possible, Zener estimates that "the tropical oceans in the year 2000 could supply the whole world with energy at a per capita rate of consumption equal to the U.S. per capita rate in 1970, and suffer only a one-degree C drop in temperature". This is estimated to be about 60 billion kilowatts. Zener proposes a thermal cycle using ammonia as the working fluid and operating between 5°C and 25°C. His estimated plant size is about 8000 cubic feet.

The cycle consists of taking high pressure liquid at 25°C, expanding it through a turbine, and condensing the vapor at 5°C. The fluid would then be pressurized and heated to 25°C to begin the cycle again. Sea water would be used to both heat and cool the fluid through the use of heat exchangers. Ammonia is suggested as a working medium.

F.2 Working Fluid

Without exploring the heat exchanger problems involved in detail, the feasibility of the turbines needed for such a cycle was evaluated. The calculations were based upon a plant output of approximately 100,000 kilowatts with an assumed turbine efficiency of 90 percent. Based upon this the flow rate for a specific fluid was calculated. Ammonia and Freon 21 were chosen as the fluids; ammonia because it was proposed by Zener, and Freon 21 because it is typical of "the recently developed refrigerating fluids" mentioned by Zener. The cycle operating points are summarized as follows, along with the necessary mass flow rates.

Point	Enthalpy (BTU/lbm)	Temp (^{o}R)	Pressure (psia)
		AMMONIA	
1	628.74	528	124.4
2	612.575	510	89.19
2S	610.779	510	89.19
		FREON 21	
1	127.56	528	22.2
2	124.323	510	15.33
2S	123.963	510	15.33

MASS FLOW RATE

Ammonia - 6050 lbm/sec

Freon 21 - 30220 lbm/sec

F.3 Range of Input

The inputs chosen for the radial turbine were based upon a back calculation from specific speed to RPM and number of units in parallel. The RPM was quantified as being 3600/N where N is an integer number. This was done because it is assumed that the machine would be used to produce 60 hertz power without benefit of a reduction gear. The number of units was then specified to provide a variety of machines covering the range of specific speed recognized as feasible, section 2.4.9. Twelve rotor blades were specified for the ammonia trials and 15 for Freon 21. This brackets the usual design range.

The axial turbine inputs were based on combinations of parameters used to achieve high efficiency. A large slow machine was favored with high reactions and few stages. This was not immediately apparent at the outset, but after several runs with no combinations yielding a feasible turbine, it was indicated.

A summary of design inputs for one run is shown below. The feasibility column indicates whether the design meets the feasibility criteria of the model, sections 2.3.8 and 2.4.9.

Axial Turbine Inputs-Ammonia

# Stages	Reaction	RPM	α_{1m}	Units	Feasible
2	.5	900	60	4	Yes

# Stages	Reaction	RPM	α_{1m}	Units	Feasible
10	.5	720	60	4	No
5	.5	720	60	4	Yes
2	.5	720	60	4	Yes
10	.5	900	60	2	No
5	.5	900	60	2	No
2	.5	900	60	2	Yes
10	.5	720	60	2	No
5	.5	720	60	2	No
2	.5	720	60	2	Yes
5	.5	900	60	4	Yes
10	.5	900	60	4	No
2	.5	720	60	10	Yes
5	.5	720	60	10	Yes
10	.5	720	60	10	Yes
2	.5	900	60	10	Yes
5	.5	900	60	10	Yes
10	.5	900	60	10	No
5	.5	900	60	4	Yes
5	.6	900	60	4	Yes
5	.7	900	60	4	Yes
5	.8	900	60	4	Yes
5	.5	900	70	4	No
5	.6	900	70	4	No
5	.7	900	70	4	Yes
5	.8	900	70	4	Yes
5	.5	900	80	4	No
5	.6	900	80	4	No
5	.7	900	80	4	No
5	.8	900	80	4	No
20	.2	720	60	20	No
20	.2	600	60	20	No
10	.2	600	60	20	Yes
20	.2	600	50	20	No
10	.3	600	60	20	Yes
20	.3	600	60	20	No
20	.3	600	50	20	Yes
20	.3	600	40	20	Yes
20	.3	600	30	20	Yes
20.	.3	600	20	20	Yes

Axial Turbine Inputs-Freon 21

2	.7	600	60	20	Yes
5	.7	600	60	20	No
10	.7	600	60	20	No
2	.7	720	60	20	No
5	.7	720	60	20	No

# Stages	Reaction	RPM	α_{1m}	Units	Feasible
10	.7	720	60	20	No
2	.7	720	70	20	No
5	.7	720	70	20	No
10	.7	720	70	20	No
2	.7	720	80	20	No
5	.7	720	80	20	No
10	.7	720	80	20	No
2	.2	720	60	20	No
2	.3	720	60	20	No
2	.4	720	60	20	No
2	.6	720	60	20	No
2	.7	720	60	20	No
2	.8	720	60	20	Yes
2	.9	720	60	20	No
2	.7	600	70	20	No
5	.7	600	70	20	No
10	.7	600	70	20	No
2	.7	600	80	20	No
5	.7	600	80	20	No
10	.7	600	80	20	No

Radial Turbine Inputs–Ammonia

RPM	Units	# Blades	Feasible
3600	13	12	Yes
3600	12	12	Yes
3600	11	12	Yes
3600	10	12	Yes
3600	9	12	Yes
3600	8	12	Yes
3600	7	12	Yes
3600	6	12	Yes
3600	5	12	Yes
3600	4	12	No
3600	14	12	Yes
3600	15	12	Yes
3600	16	12	Yes
3600	17	12	Yes
3600	18	12	Yes
3600	19	12	Yes
3600	20	12	Yes
3600	21	12	Yes
3600	22	12	Yes
3600	23	12	Yes
3600	24	12	Yes
1800	1	12	No

RPM	Units	# Blades	Feasible
1800	2	12	Yes
1800	3	12	Yes
1800	4	12	Yes
1800	5	12	Yes
1800	6	12	Yes
1200	1	12	Yes
1200	2	12	Yes
900	1	12	Yes

Radial Turbine Inputs-Freon 21

3600	250	15	No
3600	500	15	Yes
3600	750	15	Yes
3600	1000	15	Yes
3600	1250	15	Yes
3600	1500	15	Yes
3600	1750	15	Yes
3600	2000	15	Yes
3600	2250	15	Yes
3600	2500	15	Yes
3600	2750	15	Yes
3600	3000	15	Yes
3600	3250	15	Yes
3600	3500	15	Yes
3600	3750	15	Yes
3600	4000	15	Yes
3600	4250	15	Yes
3600	4500	15	Yes
3600	4750	15	Yes
3600	5000	15	Yes
3600	5250	15	Yes
3600	5500	15	Yes
3600	5750	15	Yes
3600	6000	15	Yes
3600	6250	15	Yes

F.4 Sample Output

The output for both working fluids produced many designs which could be called feasible. A representative sample of axial and radial designs for each fluid is included on the following pages. These designs are not necessarily optimums, as the object of the runs was to test the program rather

than optimize a design. Data from these runs was used to substantiate the evaluation of the model, sections 3.5 and 3.6. These examples do, however, illustrate several points about the Zener Cycle and will be the basis for the next section.

Axial Turbine-Ammonia

```
Assumed  5. Stages  0.50 Reaction at mean  720. RPM
Assumed  60.00 ALFAMN  4. Units  6050.00 ṁ

Last stage tip rel Mach no.  0.306 Last stage hub real Mach
                                                  no.  0.314
First stage tip diam. 8.51  Blade height 0.9599  # blades 50.
                                                Eff. 0.9589
Last stage tip diam. 8.84  Blade height 1.2949  # blades 38.
                                                Eff. 0.9598
Machine efficiency  0.9594
Max cent. stress  3003.50
Unit length  2.492
Aspect ratio, first stage 2.17  Last stage 2.34

Velocity Triangles
First Stage
Hub
UHN 248.3   VTHN 325.9   VX 164.2   VHN 365.0   WHN 181.7
WHR 297.7   ALFAHN 63.3   BETAHN 25.3   BETAHR -56.5
Mean
UMN 284.5   VTMN 284.5   VX 164.2   VMN 328.5   WMN 164.2
WMR 328.5   ALFAMN 60.0   BETAMN 0.0   BETAMR -60.0
Tip
UTN 320.6   VTTN 252.4   VX 164.2   VTN 301.1   WTN 177.9
WTR 360.3   ALFATN 56.9   BETATN -22.6   BETATR -62.9
Last Stage
Hub
UHN 235.6   VTHN 343.4   VX 164.2   VHN 380.6   WHN 196.4
WHR 287.2   ALFAHN 64.4   BETAHN 33.3   BETAHR -55.1
Mean
UMN 284.5   VTMN 284.5   VX 164.2   VMN 328.5   WMN 164.2
WMR 328.5   ALFAMN 60.0   BETAMN 0.0   BETAMR -60.0
Tip
UTN 333.3   VTTN 242.8   VX 164.2   VTN 293.1   WTN 187.5
WTR 371.5   ALFATN 55.9   BETATN -28.9   BETATR -63.8
```

```
Axial Turbine-Freon 21

Assumed  2. Stages  0.70 Reaction at mean  720. RPM
Assumed  60.00 ALFAMN  20. Units  30220.00 ṁ

Last stage tip real Mach no.  0.741  Last stage hub real
                                          Mach no. 0.542
First stage tip diam. 8.87  Blade height 1.9793  # blades 84.
                                          Eff. 0.9712
Last stage tip diam. 9.67  Blade height 2.7744  # blades 23.
                                          Eff. 0.9701
Machine efficiency 0.9706
Max cent. stress 5878.27
Unit length 0.802
Aspect ratio, first stage 10.00 Last stage 4.59

Velocity Triangles
First Stage
Hub
UHN 185.2  VTHN 218.7  VX 90.0  VHN 236.5  WHN 96.0
WHR 205.9  ALFAHN 67.6  BETAHN 20.4  BETAHR -64.1
Mean
UMN 259.8  VTMN 155.9  VX 90.0  VMN 180.0  WMN 137.5
WMR 275.0  ALFAMN 60.0  BETAMN -49.1  BETAMR -70.9
Tip
UTN 334.4  VTTN 121.1  VX 90.0  VTN 150.9  WTN 231.5
WTR 346.3  ALFATN 53.4  BETATN -67.1  BETATR -74.9
Last Stage
Hub
UHN 155.2  VTHN 260.9  VX 90.0  VHN 276.0  WHN 138.8
WHR 179.4  ALFAHN 71.0  BETAHN 49.6  BETAHR -59.9
Mean
UMN 259.8  VTMN 155.9  VX 90.0  VMN 180.0  WMN 137.5
WMR 275.0  ALFAMN 60.0  BETAMN -49.1  BETAMR -70.9
Tip
UTN 364.4  VTTN 111.2  VX 90.0  VTN 143.0  WTN 268.8
WTR 375.4  ALFATN 51.0  BETATN -70.4  BETATR -76.1
```

Radial Inflow Turbine-Ammonia

Assumed 3600 RPM 17. Units 12. Blades 6050.00 WW

Nozzle inlet diam. 6.99 Blade height 0.2338
Nozzle outlet diam. 4.63
Rotor inlet diam. 3.70 Outlet tip diam. 2.91
Rotor hub diam. 1.52
Unit length 1.09
Max scroll area 3.972
Specific speed 0.127727
Efficiency 0.9398

Velocity Triangles
Nozzle Inlet
V1 362.8 VT1 229.8 VM1 280.7 ALFA1 39.3
Rotor Inlet
U3 696.8 VT3 580.7 VM3 364.9 V3 685.8 W3 382.9
ALFA3 57.9 BETA3 -17.7
Rotor Outlet
Hub
U4H 287.4 V4H 241.4 W4H 375.3 BETA4H -50.0
Mean
U4M 418.1 V4M 241.4 W4M 482.8 BETA4M -60.0
Tip
U4T 548.8 V4T 241.4 W4T 599.5 BETA4T -66.3

Radial Inflow Turbine-Freon 21

Assumed 3600 RPM 1250. Units 15. Blades 30220.00 WW

Nozzle inlet diam. 2.75 Blade height 0.1046
Nozzle outlet diam. 2.04
Rotor inlet diam. 1.62 Outlet tip diam. 1.23
Rotor hub diam. 0.72
Unit length 0.45
Max scroll area 0.628
Specific speed 0.113886
Efficiency 0.9345

Velocity Triangles
Nozzle Inlet
V1 173.4 VT1 124.8 VM1 120.4 ALFA1 46.0
Rotor Inlet
U3 305.8 VT3 265.0 VM3 128.1 V3 294.3 W3 134.4
ALFA3 64.2 BETA3 -17.7
Rotor Outlet
Hub
U4H 135.1 V4H 105.9 W4H 171.7 BETA4H -51.9
Mean
U4M 183.5 V4M 105.9 W4M 211.8 BETA4M -60.0
Tip
U4T 231.8 V4T 105.9 W4T 254.9 BETA4T -65.4

F.5 Comment on the Zener Cycle

Using the sample output shown in the previous sections, it is possible to calculate a volume required for each machine. Multiplying this by the number of units gives a volume to produce 100,000 kilowatts of power. This approximate volume is 600 ft^3 for Axial with Ammonia, 1200 ft^3 for Axial with Freon 21, 700 ft^3 for Radial with Ammonia, 3700 ft^3 for Radial with Freon 21. This is interesting in that it shows ammonia to be the more advantageous of the two fluids in terms of size. This would be expected, however, in that for the cycle points defined, ammonia has a greater enthalpy change per pound than Freon. Freon has the advantage of being safer to work with, however.

A second fact is apparent in the volume consideration in that the estimated volume was in the order of one to two thousand cubic feet. Zener estimated a turbine module to be 8 x 8 x 40 feet or about 2560 cubic feet. This shows that Zener's estimation was reasonable for plants of 100,000 kilowatt output. Assuming this size plant, then to supply the output of 60 billion kilowatts mentioned in section F.1, 600,000 individual plants would be required. This also assumes that the heat exchangers are of comparable size.

Comparisons could be made for the amount of working medium required, amount of cooling water and others. That is not really the point of the evaluation, however. Exami-

nation of the sample designs does show, for at least the turbines, that the proposal is at least within the realm of possibility if not reasonability.

153105

Thesis
P7486 Potter
Preliminary turbine
design for thermal cy-
cle feasibility.